THEODOSII SPHÆRICA:

Methodo Nova Illustrata, & Succinctè Demonstrata.

Per ISAACUM BARROW,
Ex-professorem Lucasianum *Cantab.* & Soc. Regiæ Soc.

LONDINI,
Excudebat *Guil. Godbid,* vœneunt apud *Robertum Scott,*
in vico Little-Britain. 1675.

De THEODOSIO, *ex* VOSSIO *de Scientiis Mathematicis.*

In temporibus Ciceronis ac Pompeii claruit Theodosius Tripolites qui partem Geometriæ de figura Sphærica libris tribus utilissimis egregiè excoluit.

Eos Græcè edidit Latinèque vertit ac Lutetiæ Wechelii *typis mandavit* Joannes Pena *Mathematicus Regius. Compluria ex eo* Ptolomæus *hausit &* Ptolemæo *recentiores*, Pappus, Proclus, Theon, *multa item; sed, dissimulato* Theodosii *nomine; exscripsit* Vitellio, *ob quæ tamen maximè fuit admirationi.* Arabes *etiam in Linguam suam transtulerunt. Ex Arabico verò Latinè versus fuit à* Platone Tiburtino, *ut auctor est, qui de Speculis ustoriis scripsit. Eaque interpretatio* Venetiis *excusa, ante annos jam* 155 *eo nempe à quo & Almansoris Judicia ex Arabico Latine versa sunt. Sed immane quantum interest inter genuinum* Theodosium *& Arabicè loquentem, usque adeò ut ob tam multa addita alius videri possit, quæ res compulit* Joannem Penam *regium apud Parisienses Mathematicum, ut, non contentus Græcè edere, quod per se magnam laudem merebatur, ex Græco etiam optima fide faceret Latinum Parisiis editus* 1558, *& post eum* Christophorus Clavius, Herigonius, *& nuperrime* Guarini *in* Euclide *adaucto, &* Claudius Millet *de Chales in cursu Mathematico hoc opus illustrarunt. Quanquam autem*

autem Theodosius *hic* Tripolites *inscribatur, eòque nomine à* Suida *etiam memoretur: non dubium tamen quin idem sit ac* Bithynus *ille* Theodosius, *professione philosophus; quem Mathematicis disciplinis uti & filios ejus, præstitisse, ait* Strabo, *lib.* 12. *Ut* Pompeii *magni temporibus vixisse videatur. Nec obstat quod dicatur* Bithynus. *Nam fortasse ex* Bithynia, Tripolin *in* Africam *abiit, atque ibi sedem fixit. Quomodo* Hipparchus *eidem* Straboni *est* Bithynus; *qui* Ptolemæo, *& aliis, est.* Rhodius. *Scripsit etiam* περὶ οἰκήσεων, de habitationibus, *cujus Argumenti* 12 *Propositiones dedit* Mersennius *in Synopsi Mathematica, pag.* 246, *& tribus sequentibus; & libros* 2, περὶ ἡμερῶν καὶ νυκτῶν, de diebus ac noctibus. *Quæ Græcè in Bibliotheca Regia* Lutetiæ, *adservantur, Latinè* Argentorati *edi curavit* Petrus Dasypodius, *anno* 1572.

THE-

THEODOSII SPHÆRICORUM LIBER PRIMUS.

DEFINITIONES.

I.

SPHÆRA est figura solida comprehensa una superficie, ad quam ab uno eorum punctorum, quæ intra figuram sunt, omnes rectæ lineæ ductæ sunt inter se æquales.

Aliter in Elemento 11° definitur, à Semicirculi circumductu.

II.

Centrum autem Sphæræ est ejusmodi punctum.

III.

Axis verò Sphæræ est recta quædam linea per centrum ducta, & utrinque terminata in Sphæræ superficie, circa quam quiescentem circumvolvitur Sphæra.

IV.

Poli Sphæræ sunt extrema puncta ipsius axis.

V.

Polus Circuli in Sphæra, est punctum in superficie Sphæræ, à quo omnes rectæ lineæ ad circuli circumferentiam tendentes sunt inter se æquales.

Ut si omnes rectæ P Z à puncto P ad circumferentiam circuli B Z A ductæ sint æquales, erit P polus circuli B Z A: & inversè si P sit polus, erunt omnes P Z æquales inter se. Fig. 1.

VI.

In Sphæra æqualiter distare à centro sphæræ circuli dicuntur, cum perpendiculares quæ à centro sphæræ in ipsorum plano ducuntur, sunt æquales. Longiùs autem abesse ille dicitur, in cujus planum major perpendicularis cadit.

Prop. I. Theor.

Fig. 2. Si ſphærica ſuperficies (B A C) ſecetur aliquo plano (B C); linea (B G C F E), quæ fit in ſphæræ ſuperficie, eſt circumferentia circuli.

a 1. *def. hujus.* b *hyp.* c 15. *def.* 1. Nam primò tranſeat planum per ſphæræ centrum D: [a] liquet igitur rectas quaſcunque (D E, D F &c.) à D ad lineam B G C F E, [b] hoc eſt è centro ſphæræ ad ejus ſuperficiem, eductas inter ſe æquari; [c] & proinde lineam B G C F E eſſe circuli circumferentiam.

Fig. 3. d 12. 11. e 3. *def.* 11. f 47. 1. g 1. *def. huj.* h 1 *ax.* 1. & 3. *ax.* 1. k 15 *def.* 1. Sin planum non tranſeat per centrum ſphæræ, ab eo (D) [d] ducatur D H plano B C perpendicularis; & ab H ad lineam B G C F E ducantur utcunque rectæ H E, H F, & connectantur D E, D F. Eritque (ob [e] ang. D H E, D H F rectos) D Hq + HEq [f] = (DEq [g] = D Fq =) D Hq + H Fq. [h] quare H E = H F. Eodem modo rectæ omnes ab H ad lineam B G C F E eductæ æquantur: [k] ergo [i] linea B G C F E eſt circumferentia circuli. *Q. E. D.*

Coroll. Idem eſt ſphæræ centrum & circuli, qui fit plano per ſphæræ centrum trajecto.

Centrum circuli, qui fit plano non per centrum ſphæræ trajecto, eſt in perpendiculari à centro ſphæræ in planum demiſſâ.

Prop. II. Probl.

Fig. 4. Datæ ſphæræ (B A D C) centrum reperire.

a 1 *hujus.* b 1. 3. c 12. 11. d 11. 11. e 6. 11. f *cor.* 1 *huj.* Data ſphæra ſecetur utcunque plano [a] facienti circulum B E D, [b] cujus centrum F; ab F [c] erigatur F A recta plano B D E, quæ ſphæræ occurrat punctis A, C; biſecetur A C in G: erit G centrum ſphæræ. Si negas, eſto H centrum ſphæræ, [d] à quo ducatur H I recta plano B D E: [e] eſtque H I ipſi G F parallela; & punctum I eſt centrum circuli B E D, ſed F ponebatur centrum: quæ repugnant.

Coroll. Si in ſphæra ſit circulus non per centrum ſphæræ tranſiens, ſphæræ centrum erit in rectâ è centro circuli ad ipſius planum perpendiculari.

Prop. III. Theor.

Fig. 5. Sphæra (A D B) planum (E F), à quo non ſecatur, non tangit in pluribus punctis uno.

a 1 *hujus.* Tangat, ſi fieri poteſt, punctis A, B; & à ſphæræ centro C conjungantur C A, C B; planum per C A, C B [a] facit in ſphæra circulum A D B,

ADB, in plano tangenti [b] rectam EABF: [c] hæc circulum ADB secat, & proinde sphæram: ergo planum secat sphæram, non tangit, contra hypothesin. b 3. 11. c 2. 3.

Cor. Recta, quæ duo puncta in sphæræ superficie signata connectit, intra sphæram cadit.

Prop. IV. Theor.

Si sphæra (ACDE) tangat planum (AHKIL), quod eam non secet, recta linea (BA) ducta à sphæræ centro (B) ad contactum (A), perpendicularis erit ad planum (AHKIL). Fig. 6.

Nam per BA ducantur utcunque duo plana, quæ faciant in sphæra circulos ACDE, AFDG, in plano tangenti rectas HAI, KAL. Et quia sphæræ centrum B [a] est quoque centrum circulorum ACDE, AFDG, & rectæ HI, KL tangunt hos circulos ([b] quippe non secant), [c] erit BA utrique HI, KL perpendicularis; ergo & plano per HI, KL ducto recta erit. *Q. E. D.* a *cor.* 1. *huj.* b *hyp.* c 18. 3. d 4. 11.

Prop. V. Theor.

Si sphæra (ABCD) tangat planum (EF), quod ipsam non secet, à contactu autem (C) excitetur recta linea (CA) ad angulos rectos ipsi plano (EF), in lineâ excitatâ (CA) erit sphæræ centrum. Fig. 7.

Si negas punctum G extra CA sit centrum, jungatur CG: [a] ergo CG plano EF recta est. [b] ergo CA eidem plano EF recta non est, contra hypothesin. a 4. *hujus.* b 13. 11.

Prop. VI. Theor.

Circulorum (AD, BC, EF), qui in sphæra sunt, maximi sunt, qui per sphæræ centrum (G) ducuntur (AD); aliorum verò (BC, EF) illi inter se æquales sunt, qui æqualiter à centro (G) distant; qui verò longiùs à centro distant, minores sunt: & circuli in sphæra maximi per sphæræ centrum transeunt; aliorum autem æquales à centro æqualiter distant, minores verò longiùs à centro distant. Fig. 8.

Ducantur GH recta circulo BC, & GI recta circulo EF; & connectantur GD, GC, GE.

1. GC recto angulo GHC opposita [a] major est quàm CH; hoc est radius circuli AD major est radio circuli BC: ergo circulus AD major est circulo BC. a 47. 1.

2 & 3. GHq + HCq = (GCq = GEq) = GIq + IEq. ergo si GH = GI, erit HC = IE; & proinde circulus BC æqualis circulo EF: sin GH ⊏ GI, erit HC ⊐ IE, & propterea circulus BC minor circulo EE.

4. Si circulus maximus AD per centrum non transeat, alius transeat, is ergo (ut modò ostensum est) circulo AD major erit, contra hyyothesin.

5. & 6. Ob GHq + HCq = GIq + IEq; si HC = IE, erit GH = GI: sin HC ⊐ IE, erit GH ⊏ GI.

Prop. VII. Theor.

Fig. 9. Si in sphæra (BAC) sit circulus (BFCG), à sphæræ autem centro (D) ad circuli centrum (E) connectatur recta linea (DE), connexa linea (DE) ad planum circuli (BFCG) recta erit.

Ducantur BC, GF diametri circuli BFCG, utcunque & connectantur DB, DC, DF, DG. Quoniam trigona DEB, DEC sibi mutuò æquilatera sunt (quippe DB = DC, EB = EC, & DE commune est) [a] erunt anguli DEB, DEC pares, & proinde recti. Simili discursu anguli DEF, DEG recti erunt. [b] ergo DE recta est plano BFCG. *Q. E. D.*

a 8. 1.
b 4. 11.

Prop. VIII. Theor.

Fig. 10. Si in sphæra (BADC) sit circulus (BGDH), & à sphæræ centro (E) ad circulum (BGDH) ducatur perpendicularis (EF), quæ ad utramque partem producatur, cadet ea in ipsius circuli polos (A, C).

Ducantur utcunque diametri BD, GH, & connectantur AB, AD, AH, AG. Quoniam radii FB, FD, FG, FH pares sunt, [a] & anguli AFB, AFD, AFG, AFH recti, [b] erunt subtensæ AB, AD, AG, AH æquales. [c] ergo A est polus circuli BGDH. Similique argumento C ostendetur ejusdem polus. *Q. E. D.*

a 3. *def.* 11.
b 4. 1.
c 5. *def. huj.*

Prop. IX. Theor.

Fig. 10. Si sit in sphæra (BADC) circulus (BGDH) & ab altero (A) polorum ejus in ipsum ducatur perpendicularis recta linea (AFC), cadet hæc in circuli centrum (F), & inde producta cadet in reliquum polum ipsius circuli.

Nam

Nam per F ducantur rectæ B D, G H utcunque, & connectantur A B, A D, A G, A H. Et quoniam rectæ A B, A D, A G, A H [a] pares sunt, & anguli A F B, A F D, A F G, A F H [b] recti, [c] erunt rectæ F B, F D, F G, F H æquales. [d] ergo F est centrum circuli B G D H. [e] Item quia centrum sphæræ est in recta A F C, erit C ejusdem circuli alter polus. *Q. E. D.*

a 5. *def. huj.*
b 3. *d. f.* 11.
c 47. 1.
d 9. 3.
e 6. *hujus.*

Prop. X. Theor.

Si sit in sphæra circulus (B G D H); linea recta (A C) per ejus polos (A, C) ducta, ad circulum (B G D H) recta est, transitque per centrum circuli (F), & sphæræ (E). Fig. 11.

Fiat, ut in præcedenti, & connectantur rectæ C B, C D, C G, C H. Et quoniam trigona C A B, C A D, C A G, C A H sibi mutuò [a] æquilatera sunt, [b] erunt anguli C A B, C A D, C A G, C A H pares. [c] Ergo cùm rectæ F B, F D, F G, F H; tum anguli A F B, A F D, A F G, A F H æquantur. [d] quare A F recta est circulo B G D H, [e] & F est centrum ejus, & [f] proinde etiam centrum sphæræ est in A F. Quæ *E. D.*

a 5. *def huj.*
b 8. 1.
c 4. 1.
d 4. 11.
e 9. *hujus:*
f *cor.* 2. *huj.*

Schol. 1. *Theor.*

Si in sphæra sit circulus (B G D H), & ab altero (A) polorum ejus per sphæræ centrum (E) ducatur recta (A E F C) erit hæc ad circuli planum perpendicularis, & producta cadet in centrum ipsius, & in reliquum polum. Fig. 12.

Fiat, ut priùs, & connectantur E B, E D, E G, E H: & quia trigona A E B, A E D, A E G, A E H sibi mutuò æquilatera sunt, [a] erunt anguli E A B, E A D, E A G, E A H pares; [b] ergo cùm rectæ F B, F D, F G, F H, tum anguli A F B, A F D, A F G, A F H æquantur; [c] quare A F recta est circulo B G D H, [d] & F est centrum ejusdem, & C alter polus. *Q. E. D.*

a 8. 1.
b 4. 1.
c 4. 11.
d 9. *hujus.*

Schol. 2. *Theor.*

Si in sphæra sit circulus (B G D H), & à centro sphæræ (E) per centrum circuli (F) ducatur recta linea (A E F C), cadet hæc in utrumque polum circuli (B G D H).

Nam recta E F [a] perpendicularis est plano circuli B G D H. [b] ergo utrinque protracta cadet in polos circuli. *Q. E. D.*

a *hyp.* & 7. *b.*
b 8. *huj.*

Corollarium ex his.

Quatuor puncta; centrum circuli, poli ejuſdem, & centrum ſphæræ in una recta exiſtunt, in diametro ſcilicet ſphæræ ad circuli planum rectâ.

Prop. XI. Theor.

Fig. 12. In ſphæra (A B C D) maximi circuli (A C, B D) ſe mutuò ſecant bifariam.

a 6. *hujus.* b 3. 11. Commune enim centrum circulorum ([a] hoc eſt ſphæræ centrum) eſto G; erit hoc in communi ſectione [b] recta E F: ergo recta E F eſt diameter, utrumque circulum biſecans. *Q. E. D.*

Coroll. Interſectio duorum circulorum in ſphæra maximorum eſt diameter ſphæræ.

Prop. XII. Theor.

In ſphæra (A B C D) circuli (A C, B D), qui ſe mutuò biſecant, ſunt maximi.

Communis circulorum ſectio E F biſecetur in G; erit G centrum commune circulorum (quoniam E F diameter), quin & ſphæræ: nam ex G erigatur G H perpendicularis circulo A C, & G B recta circulo B D; [a] eritque centrum ſphæræ in utraq; G H, G I; ergo in communi puncto G. ergo cùm circuli A C, B D per centrum ſphæræ tranſeant, [b] erunt maximi circuli. *Q. E. D.*

a *cor.* 2. *huj.* b 6. *huj.*

Prop. XIII. Theor.

Si in ſphæra maximus circulus (A B C D) circulum quempiam (B E D) ad rectos angulos ſecet, & bifariam ſecat eum, & per polos (A, C).

Fig. 13. Ab F centro ſphæræ (ſeu circuli maximi A B C D) ducatur F G recta circulo B E D, [a] quæ communi circulorum ſectioni B D occurret in G. [b] eritque G centrum circuli B E D; ergo B D eſt diameter circuli B E D, ipſum biſecans; [c] & ipſius poli ſunt in F G protracta; ergo in circulo A B C D. *Q. E. D.*

a 38. 11. b *cor.* 1. *huj.* c 8. *hujus.*

Valet hæc Propoſitio, nec non 8, 9, 10 cum ipſarum Scholiis, etiam quando circulus B D maximus eſt, & tranſit per centrum ſphæræ.

Coroll. Hinc, ſi circulus maximus alterum non biſecet, neque per ejus polos tranſeat, is huic obliquus eſt.

Prop.

Prop. XIV. Theor.

Si in ſphæra maximus circulus (A B C D) circulum non maximum (B E D) biſecat, ad angulos rectos eum ſecat, & per polos.

Biſecetur ſectio B D in G, & à ſphæræ centro F connectatur F G. Et quia B D [a] eſt diameter circuli B E D, erit G ejus centrum. [b] ergo F G recta eſt circulo B E D, & producta [c] cadet in ejuſdem polos; proinde poli ejus ſunt in circulo A B C D. Quæ *E. D.*

[a] *hyp.* [b] *7 hujus.* [c] *8 hujus.*

Prop. XV. Theor.

Si in ſphæra maximus circulus (A B C D) eorum, qui in ſphæra ſunt, circulorum aliquem (B E D) ſecet per polos (A, C); ſecabit eum bifariam, & ad angulos rectos. Fig. 13.

Nam ducta recta A C [a] recta eſt circulo B E D, & per centra circulorum F, G tranſit. qnare 1°, circulus A B C D [b] rectus eſt circulo B E D; & 2°, communis circulorum ſectio B D eſt diameter circuli B E D, ipſúmque biſecat. Quæ *E. D.*

[a] *10 huj.* [b] *18, 11.*

Schol. 1. Theor.

Si in ſphæra maximus circulus (A B C D) tranſeat per polos (A, C) alterius cujuſpiam maximi circuli (B D); tranſibit viciſſim hic (B D) per illius (A C) polos (B, D). Fig. 14.

Nam circulus A B C D circulum B D [a] perpendiculariter ſecat; ergo viciſſim circulus B D circulum A B C D ſecat perpendiculariter, [b] adeóque per polos ipſius. *Q. E. D.*

[a] *15 hujus.* [b] *13 hujus.*

Schol. 2. Theor.

Si in ſphæra circulus (A B C D) circulum (B D) per polos (A, C) ſecet, circulus (A B C D) maximus eſt, & biſariam eum ſecat, & ad angulos rectos.

Nam recta A C [a] tranſit per centrum ſphæræ, [b] ergo circulus A B C D eſt maximus; [c] biſecátque circulum B D, eidemque rectus eſt. Quæ *E. D.*

[a] *10 hujus.* [b] *6 hujus.* [c] *15 hujus.*

Schol.

Schol. 3. Theor.

Vide fig. Prop. præc.

Si in ſphæra circulus (A B C D) circulum (B D) bifariam, & ad rectos ſecet angulos, circulus (A B C D) maximus eſt, eúmque ſecat per polos.

Nam biſectâ communi circulorum ſectione B D in G, erit G centrum circuli B D: in plano circuli A B C D ducatur G A ad B D perpendicularis; [a] erit G A plano B D recta; [b] ergo G A tranſit per polos circuli B D. [c] ergo circulus A B C D per ipſius B D polos tranſit, maximúſque proinde exiſtit. Quæ *E. D.*

a 4. *def.* 11.
b *Sch.* 2. 10 *h.*
c *Sch.* 2. 15 *h.*

Schol. 4. Theor.

Si in ſphæra ſit circulus (B D), & à polorum ejus altero (A) cadens recta (A G), ipſius plano perpendicularis, æquetur ejus ſemidiametro, circulus (B D) maximus eſt.

Nam G [a] eſt centrum circuli B D, & centrum ſphæræ eſt in rectâ A G, & alter polus C in eâdem protractâ: ergo circulus per A C eſt maximus. Eſto is A B C D ſecans circulum B D punctis B, D: & connectatur G D. Cùm igitur angulus A G D [b] rectus ſit (quoniam A G plano B D ponitur recta) erit A G. G D :: G D. G C. atqui G D = A G. ergo G C = (G D =) A G. ergo cùm centrum ſphæræ ſit in A G, erit G centrum ſphæræ. [c] ergo circulus D B eſt maximus. *Q. E. D.*

a 9 *hujus.*
b 3 *def.* 11.
c 6 *hujus.*

Prop. XVI. Theor.

Fig. 15.

Si ſit in ſphæra maximus circulus (A B), recta linea (C B), ducta ab ejuſdem circuli polo (C) ad circumferentiam, æqualis eſt lateri quadrati inſcripti maximo circulo.

Nam demiſsâ C E rectâ circulo A B, [a] erit E centrum circuli A B, ipſiúſque ſphæræ; ([b] quare circulus A C B D eſt maximus), & anguli ad centrum E recti; ideóque ſinguli arcus C A, C B, A D, B D ſunt quadrantes: quare ſubtenſa C B eſt latus quadrati maximo circulo A C B D inſcripti. *Q. E. D.*

a 9 *hujus.*
b 6 *hujus.*

Schol.

Si in ſphæra ſit circulus (A B), & ab ejus polo (C) ad circumferentiam ducta recta (C B) æquetur lateri quadrati in eo deſcripti, circulus ipſe (A B) maximus eſt. Du-

Ducatur C E recta circulo A B; [a]ergo E est centrum circuli A B; & C E [b] perpendicularis est radio E B. ergo C Eq + B Eq [c] = C Bq = [2] B Eq. quare C E = B E. [d] ergo circulus A B est maximus.

a 9 *hujus.*
b 3 *def.* 11.
c 47. 1.
d *Sch.* 4. 15.

Prop. XVII. Theor.

Si in sphæra sit circulus (A B), à cujus polo (C) in ipsius circumferentiam ducta recta linea (C B) æquetur lateri quadrati inscripti maximo circulo (C A B D), ipse circulus (A B) maximus erit.

Circulus enim C A D B, transiens per E centrum sphæræ, & rectam C B, est maximus; cujus arcus C B est quadrans; eique par C A etiam quadrans (ob rectam C A = C B); proinde sectio A B erit diameter circuli C A D B. eadémque A B diameter est circuli A B ([a]hunc enim bisecat;) ergo circulus A B est maximus. *Q. E. D.*

a 15 *hujus.*

Prop. XVIII. Probl.

Fig. 16. 17.

Lineam rectam describere æqualem diametro (A C) circuli cujuslibet (A B C D) in sphæra dati.

In peripheria circuli A B C D sume tria quælibet puncta A, B, D, junctisque rectis A B, A D, B D, [a] fac triangulum E F G triangulo A B D æquilaterum; & duc F H ad E F, & G H ad E G perpendiculares, concurrentes in H; & connectatur E H. Erit E H = A C. Ducatur C D: atque ob ang. E F H + ang E G H [b] = 2 rect. [c] erit ang F E G + ang F H G = 2 rect. [d] ergo quadrilaterum E F H G circulo inscribi potest; quare ang E H G [c] = (ang E F G [d] = ang A B D [c] =) ang A C D. item ang H G E [b] = (rect [c] =) ang C D A; & [b] latus E G = A D; [f] ergo E H = A C. *Q. E. F.*

a 22. 1.
b *const.*
c *cor.* 32. 1.
d *cor.* 22. 3.
c 27. 3.
d 8. 1.
e 3. 1.
f 26. 1.

Scholium. Theor.

Fig. 18.

Linea recta (A E) à polo (A) cujusvis circuli (B C), in sphæra, ad sphæræ superficiem ducta, quæ sit æqualis lineæ rectæ (A B) ab eodem polo ad superficiem circuli (B C) ductæ, in circuli (B C) circumferentiam cadet.

Not. hoc scholium prop. 19. *subnectendum est.*

Si negas; per A E ducatur circulus maximus A B E C, occurrens circulo B C punctis B, C; ergo ducta A B [a] = (A D [b] =) A E: [c] proinde arc. A B = arc. A E, contra 9. *ax.* 1.

a 5 *def. huj.*
b *hyp.*
c 28. 3.

Prop. XIX. Probl.

Fig. 19. 20. Deſcribere lineam rectam (E H) æqualem diametro (A C) datæ ſphæræ.

In ſuperficie datæ ſphæræ ſume duo puncta A, B pro libitu tuo. Polo A intervallo A B deſcribatur circulus B Z D, cujus diametro [a] 18. *hujus.* B D [a] fac æqualem F G; & ſuper F G fac triangulum F E G triangulo B A D æquilaterum; ipſiſque E F, E G duc perpendiculares F H, G H convenientes in H: erit ducta E H æqualis diametro ſphæræ.

Nam per A B, A C deſcribatur maximus circulus ſecans circulum B Z D in B, D; eritque ut in præcedenti E H = A C.

Prop. XX. Probl.

Fig. 21. Per duo data puncta (A, B) in ſphærica ſuperficie deſcribere circulum maximum.

Deſcribatur polo A, intervallo A G, latere quadrati maximo circulo inſcripti, circulus C D; & polo B, pari intervallo B G, deſcribatur circulus E F priori occurrens in G. Itaque circulus polo G intervallo G A tranſibit per B (quia G A = G B), eritque maximus [a] 17. *hujus.* ([a] quia G A æquatur lateri quadrati maximo circulo inſcripti). *Q. E. F.*

Si puncta A B opponantur ex diametro ſphæræ, liquet circulos quoſcunque per illa deſcriptos fore maximos.

Prop. XXI.

Fig. 22. 23. Cujuſlibet circuli (A B) in ſphæra dati polum invenire.

1. Sit primò datus circulus A B non maximus; ſume in ejus peripheria duo quælibet puncta C, D; biſeca verò arcum C A D in A, & arcum C B D in D; unde arc. A C B = arc. A D B; itaque ſi [a] 14. *hujus.* per puncta A, B ducatur circulus maximus A E B F, [a] erit in hoc polus circuli A C B; quare biſecto arcu A E B in E, erit E polus circuli A C B. *Q. E. F.*

2. Sin datus circulus A B ſit maximus, biſeca ſemicirculum A G B in G; polóque A, ſpatio G A (vel G B) duc circulum A E B F, biſectóque arcu A F B in F, erit F polus circuli A C B D. *Q. E. F.*

Prop.

Schol. 1. *Theor.*

Si in ſuperficie ſphæræ acceptum fuerit aliquod punctum (A), & ab eo puncto ad circumferentiam circuli cujuſpiam (B C) in ſphæra dati, cadant plures quàm duæ rectæ lineæ æquales (A D, A E, A F); acceptum punctum (A) polus eſt ipſius circuli (B C). Fig. 24.

Ex A ducatur A G recta circulo B C, & connectantur G D, G E, G F.

Ob angulos A G D, A G E, A G F rectos; & rectas A D, A E, A F pares, [a]erunt G D, G E, G F etiam pares; quare G eſt centrum circuli B C; atque inde A erit ejuſdem polus. *Q. E. D.*

a 47. 1.
b 8 *hujus.*

Schol. 2. *Theor.*

In ſphæra circuli (B F, C E), à quorum polis (A, D) rectæ (A F, D E) ad eorum circumferentias ductæ ſunt æquales, inter ſe æquales ſunt: & circulorum æqualium (B F, C E) æquales ſunt rectæ (A F, D E) ab eorum polis (A, D) ad circumferentias ductæ. Fig. 25.

A ſphæræ centro G ducantur G A, G D; [a]hæ tranſeunt per circulorum centra H, I; connectantur H F, I E; & G F, G E.

1. *Hyp.* Ob A F = D E, [a]erit ang A G F = ang D G E; item ang G H F = rect. = ang G I E. & G F = G E, ergo G H = G I. [b]ergo circuli B F, C E æquantur. *Q. E. D.*

2. *Hyp.* Quia circuli B F, C E pares ſunt, [b]erit G H = G I. item G F = G E, & H F = I E; [c]ergo ang A G H = ang D G E. [d]quare ſubtenſæ A F, D E pares ſunt. *Q. E. D.*

a 10 *hujus.*
a 27. 3.
b 6 *hujus.*
b 6 *hujus.*
c 8. 1.
d 29. 3.

Prop. XXII. *Theor.*

Si in ſphæra recta linea (A B) per centrum (A) ducta rectam aliquam lineam (C D) non per centrum ductam biſecet, ad angulos rectos ipſam ſecabit. Quòd ſi ad angulos rectos eam ſecet, etiam biſecabit ipſam. Fig. 26.

Nam circuli per A B, C D deſcripti, [a]maximi ſcilicet, centrum erit A; ergo ſi B C = B D, [b]erit A B ad C D perpendicularis; & ſi A B ſit perpendicularis, [b]erit B C = B D.

a 6 *hujus.*
b 3. 3.

THEODOSII SPHÆRICORUM LIBER SECUNDUS.

DEFINITIO.

IN sphæra circuli se mutuò tangere ducuntur, cùm communis sectio planorum utrumque circulum tetigerit.

Prop. I. Theor.

Fig. 27. In sphæra paralleli circuli (B F, C E) circa eosdem polos sunt.

Sint A, D poli circuli B F, quos connectat recta A D; [a] hæc recta est circulo B F, [b] ergo & circulo C E; [c] quare per polos circuli C E transit; ergo A, D sunt poli circuli C E. Unde liquet propositum.

a 10. 1 *hujus.*
b *conv.* 14. 11.
c 8. 1 *hujus.*

Prop. II. Theor.

In sphæra circuli (B F, C E) qui sunt circa eosdem polos (A, D), sunt paralleli.

Nam quia polos connectens recta A D utrique circulo B F, C E [a] recta est, [b] erunt paralleli isti circuli. *Q. E. D.*

a 10. 1 *hujus.*
b 14. 11.

Scholium. Theor.

Fig. 28. In sphæra non sunt plures circuli æquales, & paralleli, quàm duo.

Sint æquales duo circuli A B, E F, & paralleli: vis alterum C D hisce parem dari, & parallelum. Sint igitur [a] communes omnium poli G, H, quos connectat recta G H; [b] hæc per I centrum sphæræ transit, & parallelis circulis recta est; ergo ob circulos pares C D, E F, [c] erit I L = I M, contra 9. *ax.* 1.

a 1. 2 *huj.*
b 10. 1 *huj.*
c 6. 1 *huj.*

Prop.

Prop. III. Theor.

Si in ſphæra duo circuli (A B, A C) ſecent in eodem puncto (A) circumferentiam illius maximi circuli (A B C), in quo polos habent, ſe mutuò tangent illi circuli (A B, A C). Fig. 29.

Nam circulus A B C ad ambos A B, A C [a] rectus eſt, [b] quare communis horum ſectio (puta E D) circulo A B C recta eſt; ergo E D perpendicularis eſt rectis A B, A C (quæ communes ſunt ſectiones circuli A B C cum circulis A B, A C; & quæ diametri ſunt circulorum A B, A C). [d] ergo E D tangit circulos A B, A C, ergo hi ſe mutuò tangunt. *Q. E. D.*

a 15. 1. *huj.*
b 19. 11.
c 3. *def.* 11.
d 16. 3.
e *def.* 1. 2 *huj.*

Prop. IV. Theor.

Si in ſphæra duo circuli (A B, C B) ſe mutuò tangant, maximus circulus (D E) per eorum polos (D, E) deſcriptus, per eorum contactum (B) tranſibit. Fig. 30.

Si non tranſit per B, tranſeat per F; circulus igitur G F polo D, intervallo D F (majori quàm D B) deſcriptus ſecabit circulum C B in F. atqui circuli G F, C F ſe mutuò [a] contingunt (quia circulus maximus D F E tranſit per utriuſque polos). Quæ repugnant.

a 3. 2. *hujus.*

Prop. V. Theor.

Si in ſphæra duo circuli (A B, C B) ſe mutuò tangant, maximus circulus (D B) deſcriptus per unius (A B) polos (D), & per amborum contactum (B); per reliqui quoque circuli (C B) polos (E) tranſibit. Fig. 31.

Tranſeat circulus maximus D E per polos D, E; [a] hic per contactum B tranſibit: puta alterum maximum D B F tranſire per D, B. ergo cùm uterque arcus D B [b] ſit ſemicirculus, & D ſit polus circuli A B, erit B alter ejuſdem polus, ſitus in ſuâ ipſius circumferentiâ. *Q. E. A.*

a 4. 2 *hujus.*
b 11. 1 *huj.*

Prop. VI. Theor.

Si in ſphæra maximus circulus (A B) circulorum in ſphærica ſuperficie deſcriptorum aliquem (A C) tangat, tanget & alterum ei æqualem, & parallelum. Fig. 32.

Sint D & E poli circuli A C, quos nectat recta D E. Per A, D, E ducatur circulus maximus A D B E. Item polo E, per B, ducatur circulus

culus B F. Et quia D E utrique circulo A C, B F [a]recta est, [b]erunt hi paralleli. Item si ex semicirculis A D B, D B E demantur communis arcus D B, manet arcus A D = arc. B E. Unde circulus A C [c] æqualis est circulo B F. Denique quia polus circuli A B est in circulo A D E, in quo itidem polus circuli B F; [d] tanget circulus A B circulum B F. Unde liquet propositum.

a 10. 1 *hujus.*
b 14. 11.
c *sch.* 22. 1 *b.*
d 3. *hujus.*

Coroll. Hinc liquet contactus A, B diametraliter opponi.

Prop. VII. Theor.

Fig. 32. Si sint in sphæra duo æquales, & paralleli circuli (A C, B F), maximus circulus (A B), qui eorum alterum (A C) tetigerit, reliquum quoque (B F) tanget.

Nam si circulus A B non tangat ipsum B F, [a]tanget alterum saltem ipsi A C parem, & parallelum; ergo tanget tres circulos pares & parallelos. [b] *Q. E. A.*

a 6. 2. *hujus.*
b *sch.* 2. 2 *huj.*

Scholium. Theor.

Circuli (A C, B F) in sphæra paralleli, quos maximus aliquis circulus (A B) tangit, æquales inter se sunt.

Per [a]communes parallelorum polos D, E, & polos circuli A B describatur maximus circulus A F B, [b] qui per tactus A B transibit. Et ob semicirculos A D B, D B E, [c]erit arcus A D = B E. [d] ergo circuli A C, B F æquantur.

a 1 *hujus.*
b 4 *hujus.*
c 3. *ax.* 1.
d 2. *sch.* 21. 1. *hujus.*

Prop. VIII. Theor.

Fig. 33. Si in sphæra maximus circulus (A B) ad aliquem sphæræ circulum (C D) obliquus sit, tanget is duos circulos, æquales quidem inter se, parallelos autem prædicto circulo (C D), ad quem obliquus est.

Cape circuli C D polos E, F; perque hos, & polos circuli A B describatur circulus E A B; item polo E per A, & polo F per B describantur circuli A G, B H; [a] liquet circulum A B tangere circulos A G, B H; [b]hosque pares esse, & [c] parallelos circulo C D. *Q. E. F.*

a 3. 2 *hujus.*
b 6. 2 *hujus.*
c 2. 2 *hujus.*

Schol. Theor.

Si in sphæra maximus circulus (A B) circulorum aliquem (A G) tangat, obliquus erit ad alios circulos (C D) parallelos ei, quem tangit.

Nam

Nam quia circulus A B [a]non transit per polos circuli A G (tangit enim circulum A G, non bisecat); non transibit per polos circuli C D, [b]ergo obliquus est circulo C D. *Q.E.D.*

a 15. 1 *hujus.*
b 13. 1 *huj.*

Prop. IX. Theor.

Si in sphæra duo circuli (A B C D, E D F B) se mutuò secent, maximus circulus (A E C F) per eorum polos ductus, bisecabit segmenta (B A D, B C D, & B E D, B F D) ipsorum circulorum. Fig. 34.

Sint A C, E F sectiones circulorum A D C, E D F cum circulo A E C F, & B D sectio ipsorum A D C, E D F. Et quia circuli A D C, E D F circulo A E C [a]recti sunt; erit B D ([b]recta circulo A E C F, [c]adeóque) perpendicularis rectæ A C. Ergo cùm A C sit [d] diameter circuli A D C, [e] erit arc A B = arc A D, & arc C B = arc C D. Simili discursu arc E B = arc E D, & arc F B = arc F D. *Q.E.D.*

a 15. 1 *huj.*
b 19. 11.
c 3. *def.* 11.
d 15. 1 *huj.*
e 28. 3.

Schol. 1. Theor.

Si in sphæra circuli (A B C D, E B F D) se mutuò secent; circulus alius (A E C F) eorum segmenta (A B D, C B D; & E B D, F B D) bisecans, transit per eorum polos, estque maximus.

Nam ob A B + B C [a] = A D + D C; erunt arcus A B C, A D C semicirculi; ergo recta A C est diameter circuli A B C D; ergo cùm arcus A B, A D pares sint, erit recta B G D rectæ A C perpendicularis: & pari discursu recta B G D rectæ E F ostendetur perpendicularis; quare B G D [b]recta erit circulo A E C F per ipsas A C, E F ducto; [c]ergo ambo circuli A B C D, E B F D recti sunt circulo A E C F. [d] unde liquet circulum A E C F maximum fore, & per circulorum A B C D, E B F D polos transire. *Q.E.D.*

a 2. *ax.* 1.
b 4. 11.
c 18. 11.
d 3 *ſcb.* 15. 1. *huj.*

Schol. 2. Theor.

Si in sphæra duo circuli (A B C D, E B F D) se mutuò secent, maximus circulus (A F C E) bisecans duo quæcunque illorum segmenta (B A D, B E D), habens tamen arcum (A F C E) inter illa segmenta positum semicirculo inæqualem) transit per polos ipsorum, duóque reliqua segmenta (B C D, B F D) bisecet. Fig. 35.

Dic alium circulum maximum A G E descriptum iri per polos circulorum B C D, B F D; [a] hic segmenta B A D, B F D bisecabit, & proinde

a 9. 2 *huj.*

proinde transibit per puncta A, E; unde arcus A F C E [b]erit semicirculus, contra hypothesin; ergo A F C E transit per polos circulorum ABCD, EBFD; [a]adeóq; segmenta BCD, BFD bisecat. *Q.E.D.*

b 11. 1 *huj.*

Prop. X. Theor.

Si sint in sphæra paralleli circuli (A B C D, E F G H), per quorum polos (I) describantur maximi circuli (A E I G C, B F I H D); parallelorum quidem circumferentiæ (A B, E F, & B C, F G &c.) inter maximos circulos interceptæ similes sunt; maximorum autem circulorum circumferentiæ (A E, B F, C G, D H) inter parallelos circulos interceptæ, sunt æquales.

Fig. 36.

Sint A C, B D sectiones circuli A B C D cum circulis A I C, B I D; & E G, F H sectiones circuli E F G H cum iisdem A I C, B I D, [a]erunt A C, B D diametri circuli A B C D, & E G, F H diametri circuli E F G H; ergo L & K sunt centra circulorum A B C D, E F G H. Item (ob circulorum A C B, E G H parallelismum) [b]erunt E G, A C, & F H, B D parallelæ; [c] quare ang E K F = ang A L B; ergo similes sunt arcus A B, E F: eâdémque ratione arcus B C, F G similes ostendentur. Quinimo * subtensæ rectæ I A, I B, I C, I D, & [d] ideò arcus I A, I B, I C, I D æquantur; & pari modo arcus I E, I F, I G, I H æquantur; unde etiam reliqui arcus A E, B F, C G, D H pares sunt. *Q. E. D.*

a 15. 1 *hujus.*
b 16. 11.
c 10. 11.
* 5. *def.* 1 *huj.*
d 28. 3.

Prop. X I. Theor.

Fig. 37. 38. 39. 40. 41. 42.

Si in diametris (A C, D F) circulorum æqualium (A B C, D E F) æqualia circulorum segmenta (A G C, D H F) ad angulos rectos insistant, à quibus sumantur æquales circumferentiæ (A G, D H), quarum quælibet inchoata ab extremitate sui segmenti sit minor semisse circumferenriæ integri segmenti; à punctis autem (G, H) æquales circumferentias (A G, D H) terminantibus, ducantur æquales rectæ lineæ (G B, H E) ad circumferentias circulorum (A B C, D E F) primò positorum; ipsæ circulorum primò positorum circumferentiæ (A B, D E) interceptæ inter illas rectas lineas (G B, H E) & extremitates diametrorum (A C, D F) erunt æquales.

Ducantur G I recta plano A B C, & H K recta plano D E F ([a]cadent hæ in rectas A C, D F); connectantúrque A G, B I; & D H, E K; & B L, E M (ad centra L M). Et propter arc. G C = arc. H F, [b]erit ang G A C = ang H D F. Item ang G I A * = rect. = ang H K D; & A G [c] = D H. [d] Ergo G I = H K, & A I = D K; unde

a 30. 11.
b 27. 3.
c 29. 3.
* 3. *def.* 11.
d 26. 1.

THIS CHART WILL BE TH

FIRST CHART APPEARIN

AT THE END OF THIS FIL

WILL BE THE
APPEARING
OF THIS FILM.

de I L = K M). Item G B [e] = H E, & anguli G I B, H K E * ſunt recti: [f] ergo I B [g] = K E: itaque trigona B I L, E K M ſibi mutuò æquilatera ſunt; unde ang A L B = ang D M E. [h] ergo arcus A B, D E æquantur. *Q. E. D.*

e *hyp.*
f 47. 1.
g 8. 1.
h 26. 3.

Si perpendicularis à G, & H incidant in diametrorum extremitates (ut in 5, & 6 figuris) breviùs conficietur negotium ſic. Ob G A [k] = H D; & G B [e] = H E, & angulos G A B, H D E rectos. Erit quoque A B [l] = D E. [i] Ergo arc. A B = arc. D E. *Q. E. D.*

k *ſuprà.*
l 28. 3.

Prop. XII. Theor.

Si in diametris (A C, D F) circulorum æqualium (A B C, D E F) erigantur circulorum æqualium ſegmenta (A G C, D H F); & ab ipſis ſegmentis æquales circumferentiæ (A G, D H) ad extremitates ſegmentorum deſumantur, minores dimidiis ipſorum partibus; ab ipſis autem circulis æquales circumferentiæ (A B, D E) ſumantur ad eaſdem partes, quæ ſunt ad extremitates diametrorum: rectæ lineæ (G B, H E) ductæ à punctis in circumferentiis ſegmentorum ad puncta in circumferentiis circulorum, erunt æquales.

Fiat, ut in præcedenti; & quoniam (ut iſtuc oſtenſum) I L = K M, & B L = E M, & arc A B = arc D E, [a] erit ang I L B = ang K M E. [b] ergo B I = E K: ſed & G I = H K; & anguli G I B, H K E recti ſunt; [b] ergo B G = E H. *Q. E. D.*

a 27. 3.

In 5 & 6 Fig. ob arc A B = D E; & ideo rectam A B = D E, & A G = D H; & angulos G A B, H D E rectos, [b] erit G B = H E. *Q. E. D.*

b 4. 1.

Prop. XIII.

Si in ſphæra ſunt paralleli circuli (A B, C D, F G H), & deſcribantur maximi circuli (A F K, B H K), qui unum quidem (A B) parallelorum tangant, reliquos verò (C D, F G H) ſecent; circumferentiæ (A B, C D, F G; & A B, L E, M H) parallelorum, interceptæ inter eos maximorum circulorum ſemicirculos (B N P, A F O; & A M O, B H P), qui non concurrunt ſimiles erunt: maximorum verò circulorum circumferentiæ (A C, A L; B D, B E; & C F, L M; D G, E H; & A F, A M, B G, B H,) inter duos quoſcunque parallelos interceptæ, erunt æquales. Fig. 43.

* Fiant ſcilicet arc. K O = N A, & arc. K I = N B; unde quum.

NPK, & NOK sint semicirculi, erunt BNP, AFO semicirculi; & adeò reliqui AMO, BHP etiam semicirculi).

* 1. 2 *hujus.* Per parallelorum *polos I, & contactus A, B describantur circuli maximi QAIR, SBIT; estque arc CA[a] = arc AL; & arc DB
a 5 & 9. 2 *h.* [a] = arc BE; & arc CV[a] = arc VL; & arc DX[a] = arc XE. Porro, quum circulus QAIR circulo AFK, & circulus SBIT cir-
b 15. 1 *hujus.* culo BHK [b]recti sint; & arcus AI, BI minores quadrante (ob circulum ABRT non maximum); & arc AI = BI; erunt juxta 11 hujus arc AC, BE æquales. Ergo arcus AL, AC; BE, BD æquales sunt. Eodémque pacto arcus AM, AF, BH, BG æquantur. Unde & reliqui CF, LM, EH, DH æquantur. *Q. E. D.*

c 29. 3. Item ob arc CAL = arc DBE; [c] erit subtensa CL æqualis
d 28. 3. subtensæ DE, & [d] proinde arcus CVL arcui DXE; & semissis CV = DX: unde addito vel subtracto communi VD, erit arc CD =
e 10. 2 *huj.* arc VX. Ergo, cùm arcus VX [e]similis sit arcui AB, erit arcus CD eidem AB simili. Eadémque ratione arcus FG, neque non & arcus EL, HM eidem AB similes ostendentur. *Q. E. D.*

Prop. XIV. Probl.

Fig. 44. Dato in sphæra circulo (AB), qui minor sit maximo, datóque in ejus circumferentia aliquo puncto (A), per illud punctum describere circulum maximum, qui tangat datum circulum (AB).

a 20. 1 *huj.* Per A, & C (polum circuli AB) [a] describatur circulus maximus
b 17. 1 *huj.* CABE, è quo sumatur quadrans AD. Polo D per A ducatur cir-
c 3. 2 *hujus.* culus AZE; [b] hic maximus est, [c] & tangit datum AB in A. *Q. E. F.*

Prop. XV. Probl.

Fig. 45. Dato in sphæra circulo (AB), qui minor sit maximo; & dato aliquo puncto (G) in sphæræ superficie, quod sit inter datum circulum (AB), & alium ei æqualem & parallelum CD; per punctum illud datum (G) describere circulum maximum, qui tangat datum circulum (AB), maximo minorem.

a 20. 1 *hujus.* & *coroll. schol.* 20. 1. *huj.* Per E, F [a] polos parallelorum, & punctum G [a] describatur circulus maximus EAC, in quo capiatur quadrans BH; & polo E per H describatur circulus HI. Sumatur etiam quadrans GK, polóque G per K describatur circulus KL (qui certè maximus erit) secans circulum HI in L (secabit verò, quia punctum K cadit inter H, & I; ob

arcum

arcum G H minorem, & arcum G I majorem quadrante G K: illud, quia B H est quadrans; hoc quia A F D, ipsi F A F æqualis, est semicirculus; & A I ([b]=B H) est quadrans; & proinde D I est quadrans) tum per L & polos E, F ducatur maximus circulus E L F secans circulum A B in M; polóque demum L per M ducatur circulus M N: dico factum. Nam arc. L M [b] æquatur arcui H B quadranti, [c] ergo M N est maximus circulus. Item, quia circ. K L transit per polum L circuli M N, [d] transibit hic vicissim per polum illius, hoc est per G: denique quia circuli A B, G N secant maximum circulum E F, in quo polos habent, in eodem puncto M, [e] ipsi se contingent mutuò. Itaque factum.

b 2. & 10. 2 *hujus.*

c 17. 1 *huj.*

d 1 *sch.* 15. 1 *hujus.*

e 3 *hujus.*

Scholium.

Simili discursu, aliud punctum, quo circulus K L secat circulum H I, polus erit alterius circuli maximi transeuntis per G, tangentísque circulum A B.

Prop. XVI. Theor.

Maximi circuli, qui similes circumferentias parallelorum circulorum auferunt, aut per parallelorum polos transeunt, aut eundem unum parallelum tangunt. Fig. 46. 47. 48.

1°, Auferant circuli maximi A B C, D B E (ex parallelis A D C, F G) similes arcus A D, F G; & alter A B C transeat per polos parallelorum; erit maximorum intersectio B, polus parallelorum. Si negas, sit H polus ipsorum, & per H, G ducatur maximus H G I (secans ipsum A D C in I), unde arcus A I, F G [a] similes sunt. ergo cùm A D, F G similes ponantur, erunt A I, A D similes, adeóque pares. *Q. E. A.*

a 10. 2 *huj.*

2°. Auferant maximi A B C, D E F similes arcus A D, B E; & neuter per polos transeat, sed alter A B C unum B E tangat (in B); etiam D E F eundem B E tanget in E. Vis secare? [b] itaque per E describatur circulus maximus G E H. [c] ergo arcus A G, B E similes sunt; proinde verò arcus A D, A G similes erunt. *Q. E. A.*

b 14. 2 *huj.*

c 13. 2 *huj.*

3°. Auferant (ex parallelis A D C, G H) maximi A B C, D E F similes arcus A D, G H, & neuter per polos transeat, aut unum tangat parallelorum; [d] ergo circulus A B C parallelis obliquus est; [e] ergo tanget aliquem, puta (B E) ipsis A D C, G H parallelum; eundem B E tanget circulus D E F: si negas; per H, [f] describatur circulus maximus K H I tangens ipsum B E in I, [g] ergo arcus A K, G H similes, itidémque arcus A K, A D similes erunt. *Q. E. A.*

d 13. 1. *huj.*

e 8. 2 *hujus.*

f 15. 2 *huj.*

g 13. 2 *huj.*

Prop. XVII. Theor.

Fig. 49. In ſphæra paralleli circuli (A B, E F), inter quos, & parallelorum maximum (C D) æquales circumferentiæ (A C, C E) maximorum circulorum intercipiuntur, ſunt inter ſe æquales; illi verò (A B), inter quos & maximum parallelorum (C D) majores maximorum circulorum circumferentiæ (A C) intercipiuntur, ſunt minores.

Nam primò tranſeat circulus maximus A C E F D B per polos parallelorum; ergo communes ſectiones A B, E F [a] erunt diametri circulorum A B, E F. Jam ſi arc A C ([b] B D) = arc C E ([b] D F); ob arc C G D [c] = arc C H D; erit arc A G B = arc E H F; [d] unde ſubtenſæ A B, E F pares erunt, & proinde circuli A B, E F etiam pares. *Q. E. D.* Quod ſi arc A C (B D) ⊏ arc C E (D F) erit arc A G B ⊐ arc E H F, & ſubtenſa A B ⊐ E F, & circulus A B ⊐ E F. *Q. E. D.*

a 15. 1 *huj.*
b 10. 2 *huj.*
c 11. 1 *huj.*
d 29. 3.

Fig. 50. Non tranſeat ſecundò circulus maximus A C E F D B per parallelorum polos (G, H); verùm per hos, & polos circuli A C E F D B [e] deſcribatur circulus G I H K. [f] Itaque poli circuli G I H K erunt in utròque maximo circulo A C E F D B, & C D. ergo ad interſectiones C, D; [g] proinde arcus C I, C K æquantur.

e 20. 1 *hujus.*
f 1 *ſch.* 15. 1 *h.*
g 5. *def.* 1 *huj.* & 28. 3.

Jam ſi arc C A = arc C E, erit arc A I = arc E K. Item (ob ſemicirculos I K, G H) arc G I = H K; [h] ergo rectæ G A, H E pares ſunt, & [k] conſequenter circuli A B, E F pares erunt. *Q. E. D.*

h 12. 2 *huj.*
k 2 *ſch.* 20. 1 *h.*

Sin arc C A ⊏ arc C E; ſit arc C L = arc C E. Ergo parallelus per L æquabiter (uti mox oſtenſum) parallelo E F. [l] Quare parallelus A B minor erit ipſo E F. *Q. E. D.*

l 6. 1 *hujus.*

Prop. XVIII. Theor.

In ſphæra maximorum circulorum circumferentiæ (A C, E C) interceptæ inter maximum parallelorum (C D), & duos alios circulos (A B, E F) æquales & parallelos, ſunt æquales. Illæ verò (A C) quæ intercipiuntur inter majorem parallelum (A B) & maximum C D, ſunt minores.

Nam in 1ª hyp. ſi dicas eſſe A C ⊏ C E, [a] erit idcirco circ A B ⊐ circ C D: in 2ª hyp. ſi dicas eſſe A C = vel ⊏ C E, [a] erit ergo circ A B = vel ⊏ circ C D, contra hypotheſin.

a 17. 2 *huj.*

Prop.

Prop. XIX. Theor.

Si in ſphæra maximus circulus (A B C D) parallelos aliquot circulos (E F, G H, I K) in ſphærica ſuperficie deſcriptos ſecet quidem, non tamen per polos, in partes inæquales eos ſecabit, excepto parallelarum maximo (G H): de parallelorum autem ſegmentis in hemiſphæriorum uno (G Q H) interceptis, ea (L F M), quæ ſunt inter maximum parallelorum (G H) & polum conſpicuum (Q) ſunt majora ſemicirculo, reliqua verò (O P K), quæ ſunt inter maximum parallelorum & polum occultum (R) ſunt ſemicirculo minora; æqualium denique ac parallelorum circulorum (E F, I K) alterna ſegmenta (L F M, O I P; & L E M, O K P) ſunt inter ſe æqualia. Fig. 51.

1. Per polos Q, R & punctum B deſcribatur circulus maximus (Q B R; is [a] tranſit etiam per D, unde arc B H D = arc B G D: [b] item arc S F T = arc S E T, [b] & arc X K V = arc X I V. ergo arc L F M ⌐ arc L E M, & arc O I P ⌐ arc O K P.

[a] 11. 1 *huj.* [b] 15. 1 *huj.*

2. Si circulus E F = circ I K, per polos parallelorum Q, R, & polos circuli A B C D, ducatur circulus A G H: [c] iſtius circuli poli ſunt B, D: ergo arcus B A, B C æquantur. [d] Item arc B L = arc B O; ergo arc L A = arc O C. ergo arc L A M [e] (2 L A) = arc O C P (2 O C): unde ſubtenſa L M = O P. Ergo arc L E M = arc O I P. arc C E M = arc O K P. *Q. E. D.*

[c] 1 *ſch.* 15. 1 *hujus.* [d] 18. 2 *huj.* [e] 9. 2 *huj.*

Prop. XX.

Si in ſphæra maximus circulus (G L) parallelos aliquot circulos (A B, C D, E F) ſecet, non tamen per polos; de parallelorum aſſumptis circumferentiis in uno hemiſphærio, illæ (O B H), quæ propiùs accedunt ad polum conſpicuum (P) erunt majores, quàm ut ſimiles eſſe poſſint illis (N D I), quæ ab eodem conſpicuo polo longiùs abſunt. Fig. 52.

Nam ſi per P, I; & P, N ducantur circuli maximi ſecantes ipſum A B in R, S; erit arcus H B O major arcu R B S, [a] qui ſimilis eſt arcui I D N; unde liquet propoſitum.

[a] 10. 2 *huj.*

Prop. XXI.

Si in ſphæris æqualibus maximi circuli (B N D, F O H) ad maximos circulos (A B C D, E F G H) inclinentur; illæ (B N D) cujus polus Fig. 53. 54.

polus (P) ſublimior eſt ſupra planum ſubjectum, inclinatior erit; illi verò circuli, quorum poli æqualiter diſtant à ſubjectis planis, æqualiter inclinantur.

Per L, P polos circulorum A B C D, B N D; perque M, Q polos circulorum E F G H, F O H ducantur maximi circuli A L C, E M G;
a 6. 1 *hujus.* [a] interſectiones A C, B D, M I concurrunt in ſphæræ centro I. Et
b 19, & 3 *def.* 11. quoniam anguli A I D, N I D [b] recti ſunt, (quippe cùm planum
c 15. 2 *huj.* A N C [c] rectum ſit planis A B C D, B H D), [d] erit ang A I N inclina-
d 6 *def.* 11. tio plani B N D ad planum A B C D; paritérque ang E K O eſt in-
e *hyp.* clinatio circuli F O H ad planum E F G H. Cùm igitur ſit arc C P [e] ⊏ G Q, & arc P N = Q O, & proinde arc C N ⊏ G O, erit arc N A
f 27. 3. ⊐ arc O E; [f] quare ang A I N ⊐ ang E K O: *Q. E. D.*

Quòd ſi arc C P = G Q, ſimili diſcurſu liquet angulos A I N, E K O æquari. *Q. E. D.*

Facilè convertitur hoc Theorema.

Schol. 1. *Theor.*

Fig. 55. Circuli maximi (A B, C B) tangentes eundem parallelum (A C), æqualiter inclinantur ad maximum parallelorum (D E); qui verò (G H) majorem parallelum I G tangit, inclinatior eſt ad parallelorum maximum (D E); & circuli (A B, C B) æqualiter inclinati ad parallelorum maximum (D E) tangunt eundem parallelum (A C). Qui verò (G H) inclinatior eſt ad maximum parallelum, majorem parallelum (I G) tangit.

1. Per F polum parallelorum, & contactus A, C deſcribantur
a 5. 2 *hujus.* maximi circuli F A D, F C E; [a] tranſeunt hi per polos circulorum
b 15. 1 *huj.* A B, C B, [b] ſecántque adeò ipſos perpendiculariter: ergo F A eſt altitudo poli F ſupra circulum A B; & F C altitudo ejuſdem ſupra circu-
c 5. *def.* 1 *huj.* & 28. 3. lum C B; quare cùm ſit F A [c] = F C, liquet primum.

2. Per polum F, & contactum G deſcribatur maximus circulus F G E; ergo (ut priùs) F G eſt altitudo poli F ſupra circulum G H:
d 21. 2 *huj.* atqui F G ⊏ F A: [d] unde liquet ſecundum.

3. Per F, & polos circulorum A B, C B deſcribantur maximi circuli F A D, F C E; [b] hi ſecant illos perpendiculariter; ergo arcus F A,
c *hyp.* F C ſunt elevationes poli F ſupra circulos A B, C B; [c] ergo arcus F A,
d 3. 2 *huj.* F C pares ſunt. [d] itaque circulus intervallo F A, vel F C deſcriptus tanget circulos A B, C B; quod erat tertium.

4. Per F, & polum circuli G H deſcribatur circulus maximus F G E.

FGE. ergo arcus FG est elevatio poli F supra circulum GH. Atqui arc FG [c] ⊏ arc FA. ergo circ [d] tangens, polo F per G descriptus major erit quàm AC. Quod erat ultimum.

Schol. 2. *Theor.*

Circuli maximi (AB, CD) ad parallelorum maximum (DB) æqualiter inclinati, polos (E, F) habent in circumferentiâ ejusdem paralleli. Et circuli maximi (AB, CD) qui polos (E, F) habent in circumferentia ejusdem paralleli (EF), ad parallelorum maximum (DB) æqualiter inclinantur. Fig. 56.

1. Descriptis per G polum parallelorum, & per E, F polos circulorum AB, CD circulis maximis GE, GF; quia hi [a] recti sunt illis, [b] erunt arcus GE. GF pares. [c] ergo circulus polo G per E, F descriptus parallelus est circulo DB. *Q. E. D.*

a 15. 1 *hujus.*
b *hyp.*
c 2. 2 *hujus.*

2. Ob arc GE [d] = GF, [e] erunt circuli AB, CD æqualiter inclinati ad ipsum BD. *Q. E. D.*

d 5. *def.* 1 *b.* 2.
e *sch.* 21. *b.*

Schol. 3. *Theor.*

Si super circuli (ABDE) diametro AD constituatur rectum circuli segmentum (AFD), dividatur autem segmenti insistentis circumferentia (AFD) in duas inæquales partes (AF, FD); & à sectionis puncto (F) ad circumferentiam circuli primi plurimæ rectæ lineæ (FA, FI, FH, FB, FC, FD, FE) cadant; erit recta (FA) subtendens minorem partem (FA) insistentis segmenti omnium minima: quæ autem majorem (FD) subtendit, omnium maxima: reliquarum verò maximæ propinquior (FC) remotiore (FB) semper major est; duæ verò rectæ æquales (FE, FC) ab eodem puncto (D) in circumferentiam circuli (ABDE) à maxima (FD) æqualiter distantes. Fig. 57. 58.

Ducatur FG normalis plano ABDE, & connectantur GC, GB, GH, GI, GE; [a] liquet punctum G esse in recta AD; & ob FA [b] ⊐ FD, [c] erit GA ⊐ GD; [c] item GA ⊐ GI ⊐ GH &c. [c] & GD ⊏ GC ⊏ GB &c.

a 38. 11.
b *hyp.*
c 7. *vel* 8. 3.

Ergo in triangulis rectangulis FGA, FGI, FGH, [d] erit FA ⊐ FI ⊐ FH; at in triangulis rectangulis FGD, FGC, FGB, [d] erit FD ⊏ FC ⊏ FB. Demum ob GE [b] = GC, [d] erit FE = FC. Quæ *E. D.*

d 47. 1.

Schol. 4.

Schol. 4. Theor.

Fig. 59. Si in ſphæræ ſuperficie intra cujuſq; circuli peripheriam (ABCDE) ſignetur punctum (G) præter ejus polum (F); ab eo autem ad circuli circumferentiam plurimi arcus (GA, GB, GE, GC, GD) circulorum maximorum ducantur ſemicirculo minores; maximus eſt (GFA, qui per polum circuli ducitur; minimus autem, qui (GD) ei adjacet; reliquorum verò maximo propinquior (GB) remotiore (GC) ſemper major eſt; duo verò arcus (GE, GB) ab eodem maximo (GA) vel minimo (GD) æqualiter remoti inter ſe æquales ſunt.

a 15. 1 *hujus.* b 3. *ſch.* 21 *b.* Nam quoniam arcus AGD [a] perpendiculariter inſiſtit ſemicirculo ACD, [b] erit ſubtenſa GA major ſubtenſâ GB; [b] & GB ipſâ GC: ergo arcus GA major eſt arcu GB, & GB ipſo GC. & ſic in reliquis, juxta Scholium præcedens.

Schol. 5.

Fig. 60. Si in ſphæræ ſuperficie extra peripheriam cujuſq; circuli (ABCDE) ſignetur punctum (G) præter ejus polum (F); ab eo autem ad circuli circumferentiam plurimi arcus (GA, GG, GB, GE) ducantur ſemirculo minores, ſecantéſque circumferentiam circuli: maximus eſt qui (GFA) per circuli polum ducitur; reliquorum verò maximo propinquior (GB) remotiore ſemper major eſt: minimus autem eſt ille (GD) maximi, qui inter punctum (G) & circuli circumferentiam extra circulum interjicitur: reliquorum verò minimo propinquior (GH) remotiore (GI) ſemper minor eſt. Duo verò arcus (GB, GE; & GH, GK) ab eodem maximo vel minimo æqualiter remoti inter ſe æquales ſunt.

Itidem patet hoc ex Scholio penultimo, ex ſubtenſis ad arcus diſcurrendo.

Prop. XXII.

Fig. 61. Si in ſphæra maximus circulus (ABCD) unum quidem circulum (AF) tangat, alium verò ei parallelum (GBHD) ſecet, poſitum inter ſphæræ centrum, & eum circulum quem tangit maximus circulus; polus autem (E) maximi circuli fuerit inter utrumque parallelorum (AF, GBHD), deſcribantúrque maximi circuli (GL, HK, MP, NK,

NK, OL) tangentes duorum parallelorum majorem (GBHD): hi omnes erunt inclinati ad maximum circulum (ABCD), & eorum rectiſſimus quidém erit ille (HK), cujus contactus erit in eo puncto (H), in quo majus ſegmentum (BHD) paralleli majoris bifariam dividitur; humillimus verò & maximè inclinatus (GL) cujus contactus erit in eo puncto (G), in quo minus ſegmentum (BGD) bifariam dividitur; reliquorum autem illi quidem (MP, NK) qui æqualiter diſtant ab alterutro eorum punctorum (H, G) in quibus ſegmenta bifariam ſecantur, ſunt ſimiliter inclinati; qui verò (OL) contactum remotiorem habet à puncto (H), in quo majus ſegmentum biſecatur, inclinatior perpetuò eſt eo (NK), qui contactum eidem puncto propiorem habet. Poli denique maximorum circulorum erunt in uno circulo, qui & minor erit eo circulo (AF) quem tangit maximus in principio circulus, & eidem parallelus erit.

Per I polum parallelorum, & E polum circuli ABCD, ducatur circulus; is [a]biſecat ſegmenta BGD, BHD, [b]tranſitque per contactum A: ſit itaque circulus GAIEHC, fiat arcus HQ æqualis quadranti EA. ergo HQ ⊏ HI; atque Q cadit inter I, & A; itaque polo I per Q deſcriptus circulus QTR, erit parallelus circulo AF, eóque minor. Per I, & puncta contactuum ducantur maximorum circulorum arcus MIS, NIT, OIV, qui [c]tranſibunt per polos tangentium: ac ob arcus IH, IM, IN, IO, IG [d]æquales; & arcus IQ, IS, IT, IV, IR [d]etiam pares; erunt toti arcus HQ, MS, NT, OV, GR æquales; ergo cùm HQ ſit quadrans, erunt reliqui omnes etiam quadrantes; [e]ergo puncta Q, S, T, V, R erunt poli tangentium. *Q.E.D.*

a 9. 2 *huj.*
b 4. 2 *hujus.*
c 5. 2 *hujus.*
d 5 *def.* 2 *h.*
e 16. 1 *huj.*

Porro, quoniam arcus HM, HN [f]pares ſunt, erunt iſtis [g]ſimiles arcus Rφ, RΨ pares. ergo oppoſiti arcus QT, QS pares ſunt (nam [h]ductæ rectæ RQ, φS, ΨT eſſent [k]diametri circuli QTR) [k]ergo ducti arcus ET, ES pares erunt; & arcus ET ⊏ EV; & EQ maximus, ac ER minimus horum: [l]ergo circulus HK minimè inclinatur ad circulum ABCD, & GL maximè; & MP, NK æqualiter, & OL magis quàm NK. *Q.E.D.*

f *hyp.*
g 10 *hujus.*
h 15. 1 *hujus.*
k 4. *ſch.* 21. 2 *h.*
l 21. 2 *huj.*

Prop. XXIII.

Iiſdem poſitis, ſi circumferentiæ (MO, NP) circulorum tangentium (MO, NP) à contactibus (M, N) ad nodos (O, P) ſint æquales, prædicti circuli maximi (MO, NP) ſimiliter inclinati erunt. Fig. 62.

Nam per E polum circuli A B C D, & I polum parallelorum ducatur circulus maximus G A C; & per I, ac contactus M, N describantur circuli maximi I M, I N: & quoniam hi [a] per polos tangentium M O, N P transeunt, [b] insistunt ipsis M O, N P diametricè & rectò; & inæqualiter secantur in I, & arc M O [c] = N P; [d] ergo ductæ rectæ I O, I P æquantur: ergo circulus O K P polo I per O descriptus transibit per P; sunt etiam arcus M O, M Q, & arcus S O, S Q [e] æquales; itidémque arcus N P, N R, & arcus T P, T R pares sunt; ergo (ob arc M O = N P) erunt toti O M Q, P N R pares; [f] ideóque subtensæ O Q, P R; & [g] propterea arcus O S Q, P T R, & horum dimidii S O, T P pares erunt. Item arcus K O, [h] æquantur; ergo reliqui arcus K S, K T, hisque [k] similes arcus H M, H N æquabuntur: [k] ergo circuli M O, N P similiter inclinantur ad circulum A B C D. *Q. E. D.*

a 5. 2 *huj.*
b 15. 1 *huj.*
c *hyp.*
d 12. 2 *h.*
e 9. 2 *huj.*
f 29. 3.
g 28. 3.
h 10. 2 *huj.*
k 22. 2 *hujus.*

LIB.

THEODOSII SPHÆRICORUM LIBER TERTIUS.

Prop. I.

SI circulum (A C B D) inæqualiter secet recta linea (A B); super qua constituatur rectum circuli segmentum (A F B), non majus semicirculo; dividatur autem insistentis segmenti circumferentia in duas partes inæquales (F A, F B); recta linea (F B) subtendens earum minorem, minima est linearum rectarum (F G, F H, F K, &c.) ductarum ab eodem puncto (F) ad majorem partem (A C H) circumferentiæ primi circuli: rectarum verò ductarum ab eo ipso puncto ad circumferentiam (B C) interceptam inter illam minimam rectam (F B) & diametrum (C D) in quam cadit perpendicularis (F L) deducta ab illo puncto (F), semper minimæ propior (F G) remotiore (F H) minor est. Omnium autem maxima est ea (F C) quæ ab illo eodem puncto ducitur ad extremitatem ejusdem diametri. Item recta (F A) subtendens majorem circumferentiam segmenti insistentis, minima est earum (F A, F I, F K, &c.) quæ cadunt in circumferentiam (A C) interceptam inter ipsam & diametrum (C D), sempérque huic propior (F I) remotiore (F K) minor est. Si verò recta linea (A B) subjectum circulum secans sit ejus diameter, & reliqua omnia eadem sint ut suprà; recta linea (F B) subtendens minorem partem circumferentiæ segmenti insistentis, minima est rectarum ductarum ab illo eodem puncto ad primi & subjecti circuli circumferentiam; ea verò (F A) quam majorem partem circumferentiæ segmenti insistentis subtendit, maxima est. Fig. 63.

Connectantur rectæ L G, L H, L I, L K. Et quoniam L B [a] ⊐ L G ⊐ L H, erit (in triangulis rectangulis F L B, F L G, F L H) F B [b] ⊐ F G [b] ⊐ F H. Similiter, ob L A ⊐ L I ⊐ L K, erit F A ⊐ F I ⊐ F K. Et quoniam L C est [a] maxima omnium ab L ad circumferentiam A C B, [b] erit F C maxima omnium ab F ad eandem. a 7. 3. b 47. 1.

Sin arcus A C B, A D B pares ſint, patet res ex 3° Scholio 21æ ſecundi hujus.

Prop. II.

Fig. 65. Si recta linea (A B) ſecans circulum (A C B D) auferat ſegmentum (AC B) non minus ſemicirculo; ſuper ipſâ verò recta linea (AB) ſtatuatur aliud circuli ſegmentum A F B quod & ſemicirculo majus non ſit, & inclinatum ſit ad alterum ſegmentum (A D B), ſemicirculo non majus: dividatur autem inſiſtentis ſegmenti circumferentia in partes inæquales (AF, FB): recta linea (F B) ſubtendens minorem circumferentiæ partem, minima eſt rectarum omnium (F B, F G, FH, &c.) ductarum ab illo puncto (F), à quo ipſa ducitur, ad ſubjecti circuli circumferentiam illam, quæ ſemicirculo minor non eſt: & reliqua omnia, quæ in præcedenti, ſequuntur.

Ex caſu vario perpendicularis F L varii caſus emergunt.

Iterum demiſſâ rectâ F L ad planum A C B D rectâ F L, ductiſq; rectis C E L D, L B, L G, L H, L A, L I, L K; liquet fore F B ⊐ F G ⊐ F H ⊐ F C; & F A ⊐ F I ⊐ F K ⊐ F C, ut præcedenti.

Prop. III.

Fig. 65. 66. Si in ſphæra duo circuli maximi (A B C, D B E) ſe mutuò ſecent, ab eorum verò utroque ſumantur æquales circumferentiæ (B A, B C; & B D, B E) utrinque à puncto (B), in quo ſe ſecant: rectæ lineæ (A D, C E), quæ extrema puncta circumferentiarum connectunt ad eaſdem partes, æquales ſunt inter ſe.

Sint primùm omnes B A, B C, B D, B E æquales; & polo B ducatur circulus A D C E; [a] ergo ſectiones A C, D E ſunt ejus diametri; & F centrum; & radii F A, F B, F D, F E pares. Item ang. E F C [b] = ang A F D. [c] ergo E C = A D.

a. 15. 1 *hujus.* b. 11. 1. c. 4. 1.

Sin omnes non æquentur, polo B per A, & C ducatur circulus A G C H, cui occurrat utrinque productus arcus D B E in G, & H, ergo B G = B H, item B D = B E. ergo D G = E H. Atqui etiam (ſicut in priori parte) ſubtenſæ G A, C H, adeóque arcus G A, C H æquantur; quare cum circuli ſegmentum G B H [a] rectum ſit circulo A G C H; [a] & arcus G A H, G C H ſint ſemicirculi; [c] liquet rectas A D, C E æquari.

d. 28. 3. e. 12. 2 *hujus.*

Prop.

THIS CHART WI
SECOND CHART
AT THE END OF

HART WILL BE THE
D CHART APPEARING
E END OF THIS FILM

Prop. IV.

Si in ſphæra duo maximi circuli (A B C, D B E) ſe mutuò ſecent, ab eorúmque altero (A B C) ſumantur æquales circumferentiæ (B A, B C) utrinque à puncto interſectionis (B); & per puncta (A, C) terminantia æquales circumferentias ducantur duo plana parallela (A F G, C H I), quorum alterum (A F G) conveniat cum communi ſectione (K B) ipſorum circulorum, extra ſphæram, versùs prædictum punctum (B): ſit verò una illarum æqualium circumferentiarum (B A, B C) major utralibet circumferentiarum (B F, B H) in altero maximo circulo (D B E) interceptarum inter prædictum punctum (B) & utrumque planorum parallelorum: ea circumferentia (B H) quæ eſt inter illud punctum (B) & planum (C H I) quod non convenit cum communi ſectione (K B) ipſorum circulorum, major eſt quàm ea ejuſdem circuli circumferentia (B F), quæ eſt inter idem punctum (B) & planum (A F G) quod convenit cum communi ſectione circulorum. Fig. 67.

Polo B per A, C ducatur circulus A D C E, habens A G, I C ſectiones cum parallelis planis A F C, I H C; & A C, D E ſectiones cum maximis circulis A B C, D B E; unde harum interſectio (K) [a] erit centrum circuli A D C E.

Jam, ob A G, C I [b] parallelas, [c] erit ang K A M = ang K C N: item ang A K M [d] = ang C K N. & K A = K C. [f] ergo K M = K N; [g] adeóque M D = N E. Porro K B (communis ſectio circulorum A B C, D B E) [h] recta eſt circulo A D C E; & propterea [k] ang L K M rectus eſt; [l] ergo ang L M K acutus eſt, & ei æqualis H N E; angulus autem D M F obtuſus. ergo arc E H ⊐ arc D F; & ideo reciprocè (cùm æquales ſint arcus B D, B E) arc B H ⊏ arc B F. *Q.E.D.*

[a] 15. 1 *huj.*
[b] 16. 11.
[c] 28. 1.
[d] 15. 1.
[f] 26. 1.
[g] 3. *ax.* 1.
[h] 19. 11.
[k] 3. *def.* 11.
[l] 17. 1.

Obſ: quòd arc E H ⊐ arc D F patet. Nam quia M D, N E æquantur, perpendiculares ab M, N auferent æquales arcus in circumferentia D B E; quorum alter minor eſt arcu D F, ob angulum D M F obtuſum, alter major arcu E H, ob angulum E N H acutum: ergo liquet arcum D F eſſe majorem arcu E H.

Prop. V.

Si in circumferentia maximi circuli (A B C D) ſit polus (A) parallelorum; húncque circulum maximum ſecent ad angulos rectos duo Fig. 68.

duo alii maximi circuli (B D, E C); quorum alter (B D) ſit unus parallelorum, alter verò (E C) obliquus ſit ad parallelos: ab hoc autem obliquo circulo (E C) ſumantur æquales circumferentiæ (F G, G H) deinceps ad eandem partem maximi parallelorum, perque illa puncta (F, G, H) terminantia æquales circumferentias deſcribantur paralleli circuli (I K, L M, N O): circumferentiæ (I L, L N) maximi illius circuli primò poſiti inter parallelos interceptæ, inæquales erunt; ſempérque ea (I L) quæ propior fuit maximo parallelorum, remotiore (L N) major erit.

Per polum A, & punctum G ducatur maximus circulus A G P, liquet arcus G P, G F eſſe minores ſemicirculo (nam G P eſt minor quadrante A K, unde G F non ſecat G P inter G & P, & proinde G F etiam minor eſt ſemicirculo). Item G P tranſit per polos circuli I K, ergo arc G P ⊐ G F. Simili diſcurſu arc G Q ⊐ arc G H. Item recta per G ad centrum ſphæræ (hoc eſt communis ſectio circulorum A P, E C) ſecat interjectum planum circuli I K intra ſphæram; ergo eadem ſecabit planum paralleli N O extra ſphæram. Ergo, juxta præcedentem arc G P major eſt arcu G Q; quare cùm ſit arc I L = G P; & arc L N = G Q; erit arc I L major arcu L N. Q.E.D.

a 5. ſch. 21. 3 h.

b 10. 2 hujus.

Prop. VI.

Fig. 69.

Si in circumferentia maximi circuli (A B C D) ſit polus (A) parallelorum, húncque maximum circulum ad angulos rectos ſecent duo alii circuli maximi (B D, E C), quorum alter (B D) ſit unus parallelorum, alter verò (E C) ſit obliquus ad parallelos: ſumantur autem ab obliquo circulo (E C) æquales circumferentiæ (F G, G H) deinceps ad eaſdem partes maximi illius paralleli; & per puncta (F, G, H) terminantia æquales circumferentias, pérque polum (A) deſcribantur maximi circuli (A I, A K, A L): hi circumferentias inæquales intercipient de maximo parallelorum, quorum propior (K L) maximo circulo primò poſito (A B C D) ſemper erit major remotiore (K I).

a 5. 3 hujus.

b 10. 2 hujus.

c 3. 2 hujus.

d 15. 1. hujus.

Per puncta F, G, H deſcribantur paralleli M N, O P, Q R; a ergo arc O M ⊏ O Q; & b proinde arc G V ⊏ G X: fiat arc G Y = G X, & per Y deſcribatur parallelus S T; cùm igitur arcus G Y, G X, & arcus G H, G F pares ſint, c erunt ductæ rectæ H X, Y F æquales. Porro circulus S Z T d circulo A I recto inſiſtit, & biſecatur ab eo (ductâ ab Z ad alteram ſectionem rectâ) & ſegmentum circuli A I ab Z per I ad alteram ſectionem majus eſt ſemicirculo; &

arcus

arcus Y Z eſt quadrante minor, (nam quia circulus A B C D rectus eſt circulis B D, E C, tranſibit per eorum polos, & [c] biſecabit eorum ſegmenta; unde arcus B φ, E φ erunt quadrantes; quare K I quadrante minor eſt; & proinde ipſi [f] ſimilis Y Z quadrante minor eſt). [g] ergo recta Y Z eſt minima cadentium in circumferentiam Z I; adeóque minor ipſâ Y F, [h] hoc eſt ipſâ H X. ergo cùm circulus Q R minor ſit circulo S T, erit arcus H X major quàm ut ſit ſimilis arcui Y Z. (Major enim ſubtenſa ex minore circulo majorem proportione arcum aufert, quàm minor ex majore,) ergo arcus L K ([f] ſimilis arcui H X) major eſt, quam ut ſimilis ſit arcui K I (qui ſimilis eſt arcui Y Z). quare cùm arcus L K, K I ſint in eodem circulo, erit ſimpliciter arc L K major arcu K I. *Q. E. D.*

c 9. 2 *hujus.*
f 10. 2 *hujus.*
g 1 *hujus.*
h *priùs.*

Prop. VII.

Si in ſphæra maximus circulus (A B C D) tangat aliquem ſphæræ circulum (A E); alius autem maximus circulus (G H) ad parallelos obliquus ſit, tangátque circulos majores illis, quos tangit maximus circulus primò poſitus; fuerintque eorum contactus (G, H) in maximo circulo primò poſito; & ſumantur à circulo obliquo circumferentiæ (I K, K L) æquales, & continuæ ad eaſdem partes maximi (B D) parallelorum: per puncta autem (I, K, H) terminantia æquales circumferentias deſcribantur paralleli circuli (M N, O P, Q R); hi circumferentias inæquales (M O, O Q) intercipient de maximo circulo (A B C D) primò poſito; quarum ea (M O) quæ propior erit maximo parallelorum, erit major remotiore (O Q). Fig. 70.

Per S polum parallelorum, & punctum K deſcribatur circulus maximus S K, ſecans parallelos punctis T, V: item per K [a] deſcribatur circulus maximus K E tangens parallelum A E in E verſùs partes G (unde K E cadit inter S V, & G I; nam ſi extra G I caderet, non tangeret circulum A E, quoniam ipſi K G non priùs occurrit quàm in puncto quod opponitur puncto K). ergo cùm arcus S V tranſeat polum circuli M N, [b] erit K V minimus arcus omnium à K ad M N cadentium, & K Y minor quàm K I; paritérque K X ⊐ K L; ergo (ut in 5tâ hujus) erit arcus K Y major arcu K X; ſed arcus M O [c] æquatur arcui K Y, [c] & arcus O Q arcui K X; ergo arcus M O major eſt arcu O Q. *Q. E. D.*

a 15. 2 *h.*
b 5 *ſch.* 21. 2 *h.*
c 13. 2 *huj.*

Prop.

Prop. VIII.

Fig. 71. Si in ſphæra maximus circulus (A B) tangat aliquem ſphæræ circulum (A C;) aliquis autem alius maximus circulus (D E) obliquus ad parallelos tangat circulos majores illis, quos tangebat maximus circulus primò poſitus; fuerítque eorum contactus in maximo circulo primò poſito: ſumantur autem de obliquo circulo æquales circumferentiæ continuæ (F G, G H) ad eaſdem partes maximi parallelorum, pérque puncta (F, G, H) terminantia æquales circumferentias deſcribantur maximi circuli (M N, K L, C I), qui & tangant eundem circulum (A C) quem tangebat maximus circulus primò poſitus, & ſimiles parallelorum circumferentias intercipiant; habeántque eos ſemicirculos, qui tendunt à punctis contactuum (C, K, M) ad puncta (F, G, H) terminantia æquales obliqui circuli circumferentias, per quæ deſcribuntur, ejuſmodi ut minimè conveniant cum illo circuli maximi primò poſiti ſemicirculo, in quo eſt contactus (A) obliqui circuli inter apparentem polum, & maximum parallelorum. Inæquales intercipient circumferentias (I L, L N) de maximo parallelorum (B D), quorum propior (I L) circulo maximo (A B) primò poſito, ſemper erit major remotiore (L N).

Per puncta F, G, H deſcribantur paralleli P F, Q G, R H ſecantes circulum K L in O, S; [a] ergo arc Q P ⊏ Q R, & [b] ideo arc G O ⊏ G S; fiat arc G T = G S, & per T deſcribatur parallelus V T ſecans circulum M N in X: Jam circulus M N ſecans parallelum V X non per polos aufert ſegmentum, ab X per V ad alteram ſectionem, minus ſemicirculo; etiam, ſegmentum circuli M N ab X per N ad oppoſitam ſectionem eſt majus ſemicirculo. Item ſegmentum V X inclinatur ad M X versùs partes R. (Nam circulus B E [c] rectus eſt circulo N Y per polum incidenti parallelorum; ergo B E inclinatur ad circulum M N, ergo V X ad eundem N M inclinatur); denique ſegmentum paralleli X V, ab X per V ad alteram ſectionem, *ſecatur inæqualiter in T: è quibus recta T X [d] minor eſt rectâ T F, [e] hoc eſt rectâ H S; unde, ſicut in 6tâ hujus, erit arcus I L major arcu L N. Q. E. D.

a 7. 3 hujus.
b 13. 2 hujus.
c 15 1 huj.
* Vid. Not.
d 2. 3 hujus.
e 3. 3 hujus.

Fig. 72. *Not.* Quòd ſegmentum ab X per V ad alteram ſectionem protenſum, ſecetur inæqualiter in T, ita patebit. Ducatur circulus maximus E Z tangens parallelum A C in Z; & quoniam circulus maximus Z Y [f] tranſit per polos circulorum E Z, & B E, [g] biſecabit is horum ſeg-

f 12 hujus, & conſt.
g 9 2 huj.

ſegmenta; ergo arcus E Z eſt quadrans; paríque de cauſa arcus E D eſt quádrans; ergo circulus polo E per Z deſcriptus tranſit per Y, & D.

Simili diſcurſu N M eſt quadrans, & circulus maximus polo N per M deſcriptus tranſibit per Y, [f] pérque polos tangentis N M; [g] ergo circulus M Y biſecabit ſegmenta tangentis N M, & circulorum E B, X Y; ergo ſecabit circulum E B ad intervallum quadrantis ex N; hoc eſt ultra quadrantem ex E, hoc eſt extra circulum D B. ergo ſegmentum ab X per V ad alteram ſectionem biſecatur extra V; & arcus X V minor eſt ſemiſſe iſtius ſegmenti, magíſque arcus T X eſt minor ejuſdem ſemiſſe.

Lemma Probl.

Propoſitis duabus inæqualibus magnitudinibus (A B, A C,) reperire aliam mediam, quæ datæ cuicunque magnitudini (D G) commenſurabilis ſit. Fig. 73.

Biſecetur D G, & ejus ſemiſſis biſecetur, ac ita continuò [a] donec aliqua pars D F ſit minor quàm C B; & ſit E multiplex ipſius D F proximè major quàm A C; ergo E ⊐ A B: (nam ſi æqualis eſſet, poſſet detrahi una magnitudo æqualis ipſi D F, ſic ut ſupereſſet multiplex ipſius D F major quàm A C, contra conſtructionem), atqui E & D G commenſurabiles ſunt, [b] propter D F communem menſuram: ergo liquet propoſitum. a 1. 10. b *conſtr.*

Prop. IX.

Si polus (A) parallelorum ſit in circumferentia maximi circuli (A B), quem duo alii maximi circuli B C, D C ad rectos angulos ſecent; quorum circulorum alter (B C) ſit unus parallelorum, alter verò (D C) ad parallelos obliquus ſit: hoc obliquo circulo ſumantur æquales circumferentiæ (E F, G H) quæ continuæ quidem non ſint, ſed tamen ſint ad eaſdem partes maximi illius paralleli: per polum autem (A) & ſingula puncta (E, F, G, H) æquales circumferentias terminantia deſcribantur maximi circuli (A E I, A F K, A G L, A H M): inæquales intercipient circumferentias; quarum ea (M L) quæ propior erit maximo circulo primùm poſito, ſemper erit major remotiore (K I). Fig. 74.

Sit primùm arcus intermedius F G commenſurabilis arcui E F, vel G H; & [a] dividantur arcus E F, F G, G H in partes æquales communi menſuræ: pérque diviſionum puncta Q, P, N, & polum A ducantur circuli maximi A Q V, A P T, A N R; ergo arc M R [b] ⊏ b 6. 3. *huj.* a 4. 10.

F RL

R L ⊏ L S, & ſic continuò; ergo arc M L ⊏ K I. Q. E. D.

Fig. 75. Sit ſecundò arc F G incommenſurabilis utrique arcui E F, F G; tum ſi arcus M L non ſit major arcu K I; ſit primò minor, & ſumatur K N æqualis ipſi M L; & ducatur circulus maximus A N ſecans circulum

c *lem.* 8.3.*b.* C D in O; tum [c] capiatur arcus F P major quàm F O, & minor quàm F E, ipſique F G commenſurabilis; & huic æqualis ſit G Q, ducantúrque maximi circuli A P R, A Q S. Hinc ob arcus P F, G Q æquales, & commenſurabiles intermedio F G, erit (ut modò oſtenſum eſt) arc S L ⊏ arc K R ⊏ arc K N. Ergo arc M L ⊏ arc K N:

d *ſuppoſ.* atqui arc M L [d] = arc K N, quæ repugnant.

Fig. 76. Quòd ſi dicatur arcus M L æqualis arcui K I, biſecentur arcus E F, G H in N, O; ducantúrque maximi circuli A N P, A O Q; [c] ergo

c *6. 3 huj.* arc M Q ⊏ arc Q L; [c] & arc K P ⊏ P I; unde Q L ⊐ ½ M L (½ K I); & K P ⊏ ½ K I. quare arc Q L ⊐ K P. atqui, cùm arc G O = F N, non erit Q L ⊐ Q P, ut modò oſtenſum eſt; ergo malè ponitur arc M L æqualis arcui K I.

Igitur arcus M L non eſt minor arcu K I, nec æqualis ei; ergo major. Q. E. D.

Scholium.

Planè ſimili diſcurſu, quæ de arcubus continuis oſtenſa ſunt in propoſitionibus 5, 7, 8, etiam de non-continuis oſtendi poſſent. Valeant igitur.

Prop. X.

Fig. 77. Si polus (A) parallelorum ſit in circumferentia maximi circuli (A B), quem duo alii maximi circuli (B D, C D) ad angulos rectos ſecent; quorum alter (B D) ſit unus parallelorum, alter verò (C D) ſit obliquus ad parallelos: in hoc autem obliquo circulo (C D) ſumantur duo quælibet puncta (E, F) ad eaſdem partes maximi illius paralleli; pérque polum (A) parallelorum, & per utrumque illorum punctorum deſcribantur maximi circuli (A G, A H): erit ut circumferentia (B H) maximi parallelorum intercepta inter maximum circulum primò poſitum, & proximum maximum circulum per polum & per unum punctorum deſcriptum, ad circumferentiam (C F) obliqui circuli inter eoſdem circulos interceptam; ita circumferentia (H G) maximi parallelorum intercepta inter duos maximos circulos per polum pérque utrumque punctorum deſcriptos ad circumferentiam aliquam, quæ ſit minor, quàm circumferentia (F E) obliqui circuli inter utrumque punctum intercepta.

Sint

Sint primùm arcus C F, F E commensurabiles, & dividantur in arcus æquales communi mensuræ, perque puncta divisionum & polum A ducantur circuli maximi I M, K N, L O.

Quum igitur arcus C L, L K, K F, F I, I E æquentur; sitque ideò arc B O [a] ⊏ O N [a] ⊏ N H, & sic porro: [b] erit arc B O. C L ⊏ O N. L K ⊏ N H. K F. [c] ergo compositè arc B O + O N + N H ad arc C L + L K + K F. ⊏ arc N H. K F; hoc est arc B H. C F ⊏ arc N H. K F [b] ⊏ H M. F I, verùm arc H M. F I ⊏ H G. F E ([c] ob H M. F I ⊏ M G. I E) ergo arc B H. C F ⊏ arc H G. F E. Sit arc B H. C F :: H G. P. quare H G. P ⊏ H G. F E. [d] ergo P ⊐ arc. F E. *Q.E.D.*

a 6. 3 *hujus.*
b 8. 5.
c 34. 5.
d 10. 5.

Quòd si arcus C F, F E sint incommensurabiles: sit primò B H. C F :: H G. F I. & arcus F I major (si fieri potest) arcu F E: tum inter F I. F E medius arcus F K [e] sit ipsi C F commensurabilis; ac per polum A & K ducatur maximus circulus K L; ergo è mox ostensis, est B H. C F (hoc est H G. F I) ⊏ H L. F K. [f] ⊏ H G. F K. [g] ergo F I ⊐ F K. *Q.E.A.*

Fig. 78.
e *lem.* 8. 3 *huj.*
f 8. 5.
g 10. 5.

Sin dicatur arc B H. C F :: H G. F E; bisecetur arcus F E in X, perque polum A & X ducatur circulus maximus X Y; [h] ergo arc H Y ⊏ Y G; adeóque H Y ⊏ ½ H G. [k] ergo H Y. F X ⊏ ½ H G. F X (½ F E) :: H G. F E. ergo H Y. F X ⊏ B H. C F; [l] unde H Y ad arcum majorem arcu F X se habebit ut B H ad C F; quod fieri non posse modo demonstratum est.

Fig. 79.
h 6. 3 *hujus.*
k 8. 5.
l 10. 5.

Igitur potius ut B H ad C F, ita erit H G ad arcum minorem ipso F E. *Q. E. D.*

Coroll. arc. B H. C F ⊏ H G. F E. & permutatim.

Prop. XI.

Si polus (A) parallelorum sit in circumferentia maximi circuli (A B), quem duo alii maximi circuli (B C, D E) ad angulos rectos secent; quorum alter (B C) sit unus parallelorum, alter verò (D E) sit obliquus ad parallelos, alius autem maximus circulus (A E) per polos parallelorum transiens obliquum circulum secet (in E) inter maximum parallelorum, & eum (D F) quem obliquus circulus (D E) tangit: diameter sphæræ (D L) ad diametrum (D M) ejus circuli, quem tangit obliquus circulus, majorem rationem habet, quàm circumferentia (B C) maximi parallelorum intercepta inter maximum circulum primò positum, & maximum circulum per polos paralle-

Fig. 80.

lorum tranſeuntem, ad circumferentiam (D E) obliqui circuli inter eoſdem circulos interceptam.

Sint A G, B G, D L, D M communes ſectiones circuli A B cum circulis A C, B C, D E, D F; item C G, F N communes ſectiones circuli A E, cum circulis B C, D F. Ex polo A per E deſcribatur parallelus O E; ſintque O H, E H, E I communes ejus ſectiones cum circulis A B, A C, D E. Quoniam igitur A G [a] recta eſt plano paralleli O E, caditque in ejus centrum, erit ang G H I rectus; [b] ergo I G ⊏ I H: Fiat I K = I H, & connectatur E K; ergo (cùm circulus uterque D E, O E rectus ſit circulo A B, ac [c] ideò communis ipſorum ſectio E I recta plano A B; ideóque anguli E I H, E I K recti ſint, & latera E I, I H æquentur lateribus E I, I K) [d] erunt anguli I H E, I K E æquales. Jam D L. D M [e] :: D G. D N :: I G. I H [f] :: I G. I K * ⊏ ang I K E ([h] hoc eſt ang. I H E, [k] vel B G C). I G E. Eſt verò ang B G C. ang I G E (vel D G E) [l] :: arc B C. D E. ergo D L. D M ⊏ arc B C. D E. *Q. E. D.*

a 10. 1 *huj.*
b 19. 1.
c 19. 11.
d 4. 1.
e 15. 5.
f 4. 6.
g 7. 5.
* *Vid. Not.*
h *priùs.*
k 10. 11.
l 33. 6.

Fig. 81.

Not. Quòd I G. I K ⊏ ang I K E. I G E, ſic patet.

Ducatur G X ad K E parallela, cui occurrat I E protracta in X. centróque G per E ducatur arcus circuli Z E Y, ipſis G I (protractæ) & G X occurrens punctis Z. Y. Eſtque X E. E I [m] :: triang X G E. triang E G I [n] ⊏ ſector Y G E. triang E G I [n] ⊏ ſector Y G E. Sector E G Z [o] :: ang X G E. ang E G I. [p] ergo componendo X I. E I ⊏ ang X G I. ang E G I. [q] hoc eſt G I. K I ⊏ ang E K I. ang. E G I. *Q. E. D.*

m 1. 6.
n 8. 5.
o 33. 6.
p 28. 5.
q 4. 6.

Scholium.

Iiſdem poſitis, diameter ſphæræ ad diametrum paralleli (G E) per punctum (E) obliqui circuli, per quod maximus circulus (A C) è polo tranſit, deſcripti, minorem rationem habet, quàm circumferentia (B C) maximi parallelorum intercepta inter maximum circulum primò poſitum, & maximum circulum per polos parallelorum tranſeuntem, ad circumferentiam (D E) obliqui circuli inter eoſdem circulos interceptam.

Nam quoniam uterque circulus B C, D E tranſit [a] per polos circuli A B, erit iſtorum interſectio Q polus circuli A B; ergo ſegmentum D E L (rectò inſiſtens circulo A B) dividitur inæqualiter in E; [b] ergo ducta recta E D minor eſt ductâ E O; ergo (cùm circulus E O minor ſit circulo E D) erit arcus E O major arcu E D; [c] ergo arc B C. E O ⊐ B C. E D; ergo tota peripheria circuli B C ad totam periph. circuli E O (hoc eſt diameter B R ad diametrum O D) minorem rationem habet quàm B C ad E D. *Q. E. D.*

a 13. 1 *huj.*
b 3 *ſch.* 21. 2 *h.*
c 8. 5.

Prop.

Prop. XII.

Si in ſphæra maximi circuli (A B, C D) tangant unum eundémque parallelorum (A C), intercipiántque ſimiles parallelorum circumferentias (P K, B D) inter utrumque maximorum circulorum interjectas; alius autem maximus circulus (E F) ad parallelos obliquus circulos tangat majores (B G) illis, quos tangunt maximi circuli primò poſiti; ſecétque obliquus idem circulus eoſdem maximos circulos primò poſitos in punctis (I, K) poſitis inter maximum parallelorum, & circulum quem tangunt circuli maximi primò poſiti: diameter ſphæræ ad diametrum circuli (E G), quem tangit obliquus circulus, majorem rationem habet, quàm circumferentia (B D) maximi paralleli (H F) intercepta inter circulos (A B, C D) primò poſitos, eundémque circulum tangentes, ad circumferentiam (I K) obliqui circuli inter eoſdem circulos interceptam. Fig. 82.

Per L polum parallelorum & puncta E, I, K deſcribantur maximi circuli L H, L M, L N; & per K parallelus K O ſecans A B in P: quum igitur [a]ſit ratio diametri ſphæræ ad diametrum circuli E G major quàm ratio arcûs H M ad E I, [b] & hæc major ratione arcûs M N ad I K; ſitque arcus B D minor arcu M N (nam arcus P K, [c]ſimilis arcui B D, minor eſt arcu O K, [d] qui ſimilis eſt arcui M N); [e]erit ratio diametri ſphæræ ad diametrum circuli E G major ratione arcûs B D ad arcum I K. *Q. E. D.*

a 11. 3. *hujus.*
b *cor.* 10. 3. *hu.*
c *hyp.*
d 10. 2 *hujus.*
e 8. 5.

Prop. XIII.

Si in ſphæra paralleli circuli (C D, E F) intercipiant circumferentias (G C, G F) maximi alicujus circuli (A F) utrinque æquales ab illo puncto (G) in quo ipſe maximus circulus (A F) ſecat maximum parallelorum (B G); per puncta verò (C, F) terminantia æquales circumferentias, & per parallelorum polos deſcribantur maximi circuli; aut ſi deſcribantur maximi circuli, qui unum eundémque parallelorum tangant, æquales intercipient circumferentias (G H, G I) de maximo parallelorum. Fig. 83. 84.

Ob arcus G C, G F pares, [a]erunt paralleli C D, E F pares. [b] ergo arcus G K, G L æquantur; [c]ergo ductæ ſubtenſæ C K, F L æquales erunt; [d] unde arcus C K, F L æquantur; ergo, quum arcus G H ſimilis ſit arcui C K, [e] & arcus G I arcui F L, erunt arcus G H, G I ſimiles, & proinde æquales. *Q.E.D.*

a 17. 2 *huj.*
b 18. 2 *huj.*
c 3. 3 *huj.*
d 28. 3.
e 10, *vel* 13. 2. *huj.*

Coroll.

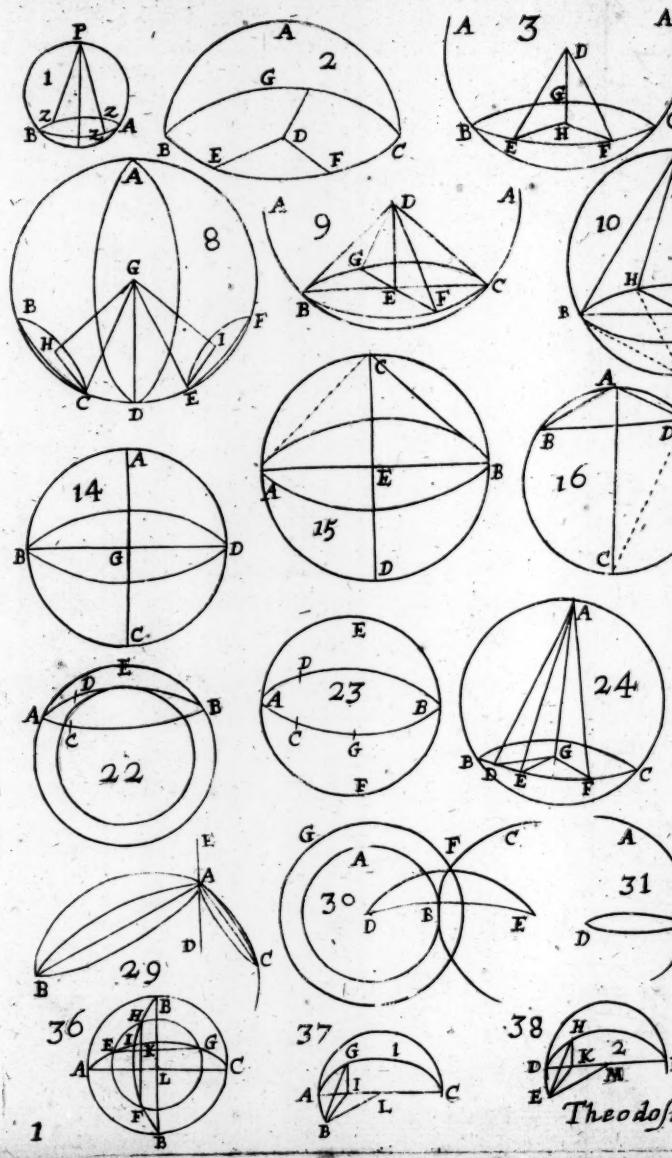
Coroll. Hinc arcus C H, H E, & ſimiliter intercepti æquales ſunt. Nam rectæ C H, H E æquantur.

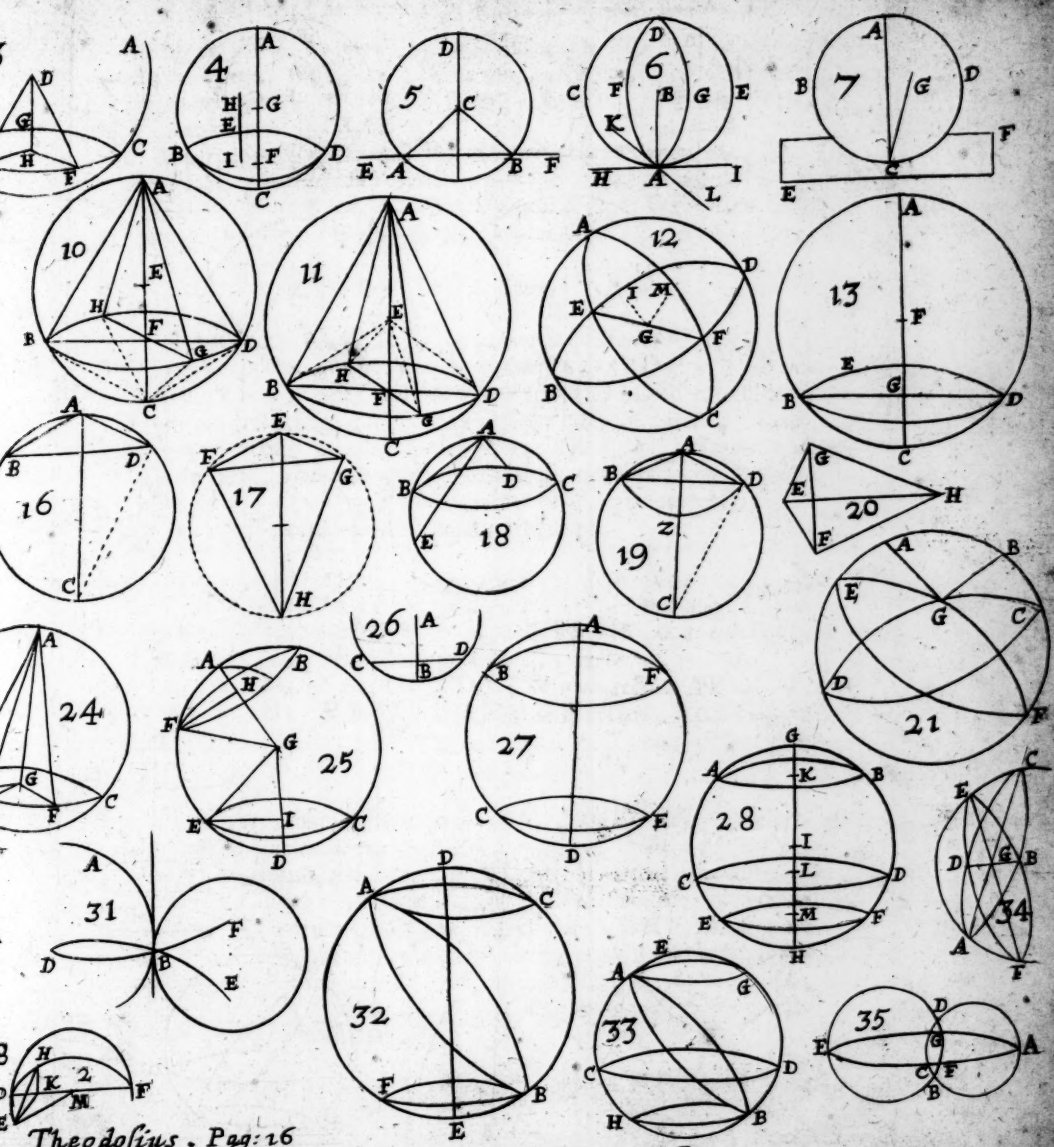
Prop. XIV.

Si in ſphæra maximus circulus (A B) aliquem circulum (A C) tangat; alius autem maximus circulus (D E) obliquus ad parallelos tangat circulos (D F) majores illis, quos tangebat maximus circulus (A B) primò poſitus; inæquales intercipient circumferentias (K H, E I) parallelorum circulorum, quorum propiores (K H, vel B E) utrivis polorum majores erunt, quàm ut ſimiles ſint remotioribus (E I, vel G K).

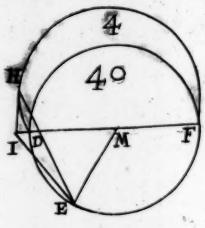
a 15. 2 *huj.* b 13. 2 *huj.* Per puncta E, K [a] deſcribantur maximi circuli L E, C K tangentes circulum A C in C, & L: [b] ergo arcus M H, E I ſimiles ſunt; quare K H major eſt, quàm ut ſimilis ſit arcui E I; paritérque ſimiles ſunt arcus B N, G K, ergo B E (propior alteri polo) major eſt quàm ut ſimilis ſit arcui G K. unde liquet propoſitum.

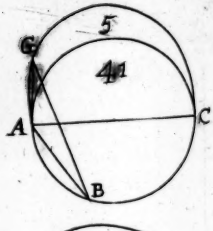
FINIS.

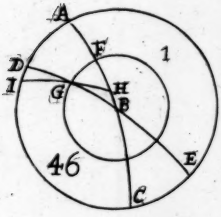

1
2
3
8
9
10
14
15
16
22
23
24
29
30
31
36
37
38
Theodos

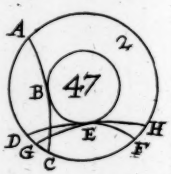


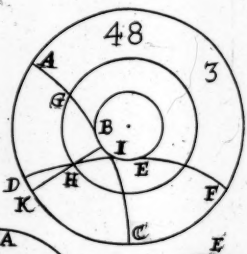
Theodosius. Pag: 16

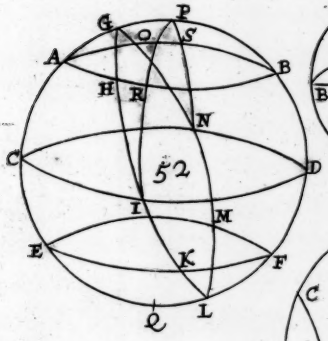


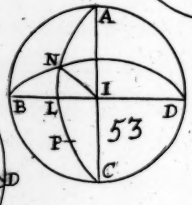


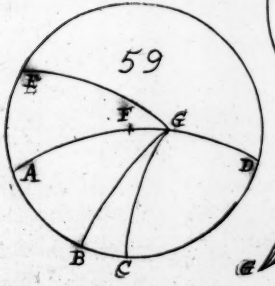


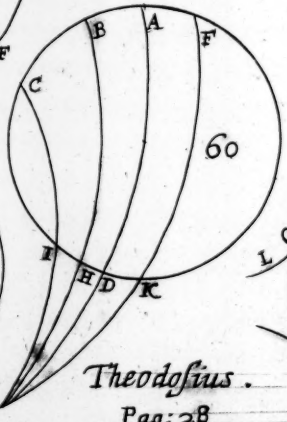




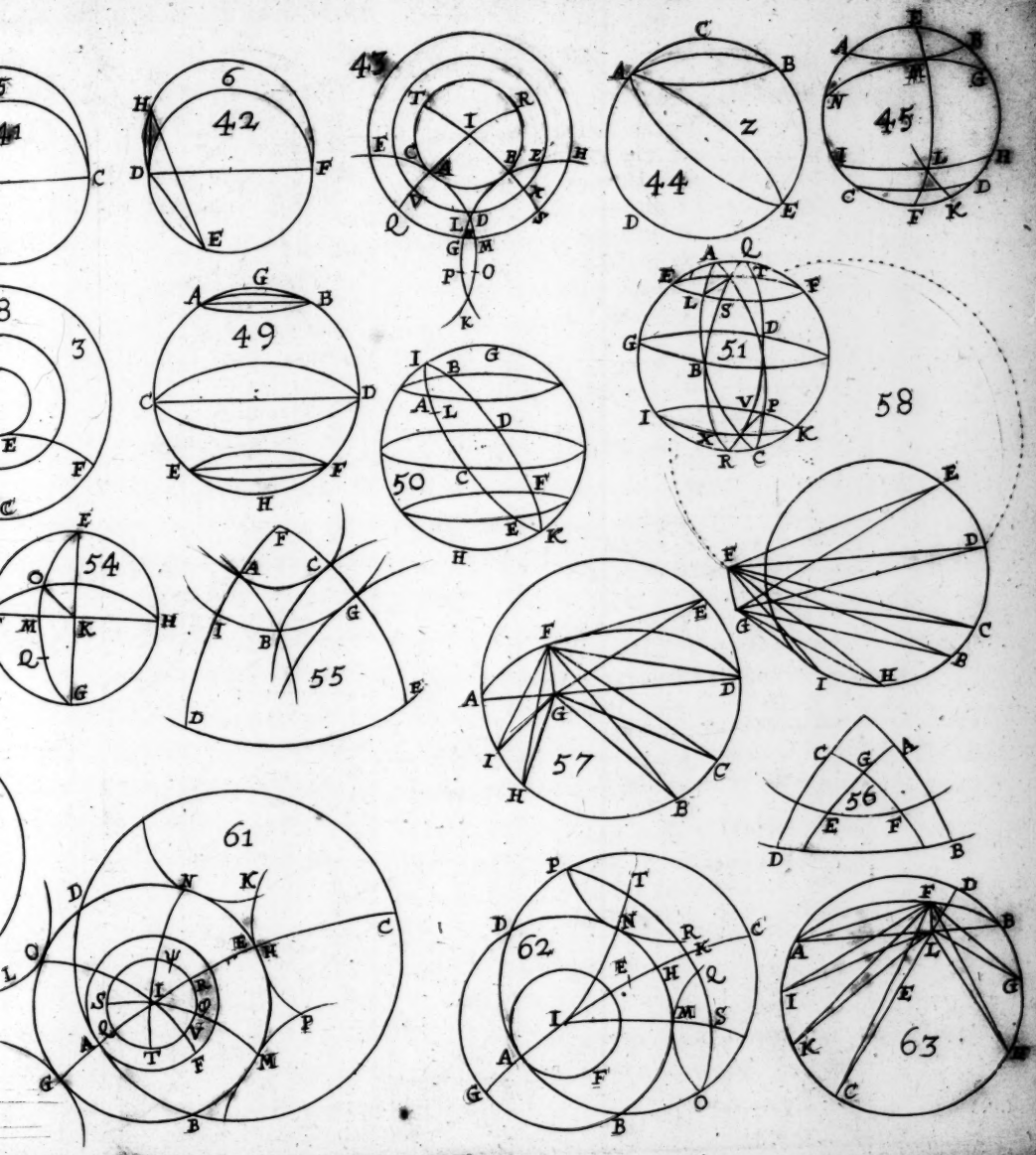


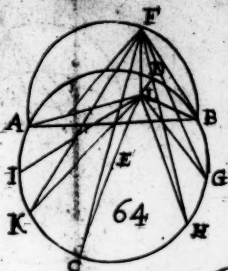


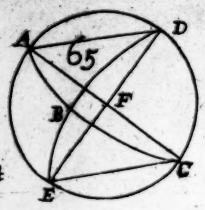




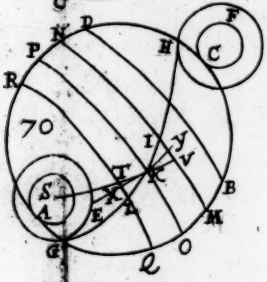
Theodosius.
Pag: 28

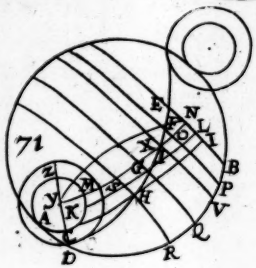

41
42
43
44
45
49
50
51
54
55
56
57
58
61
62
63

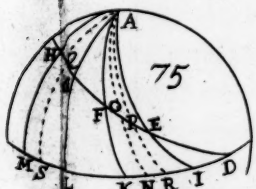

64

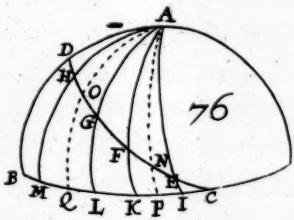

65

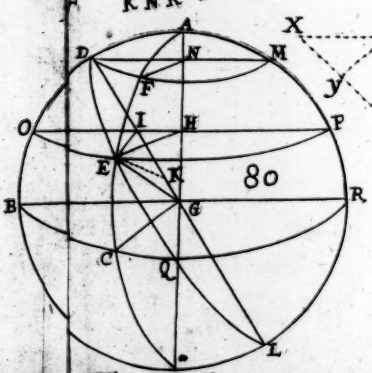
66

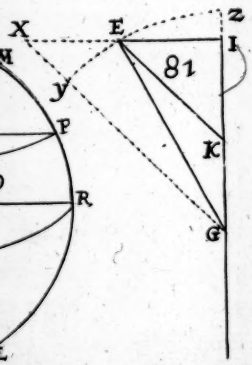

70


71

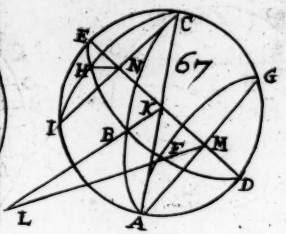

75

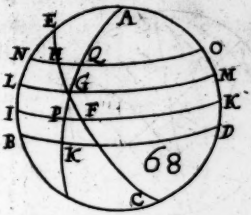

76

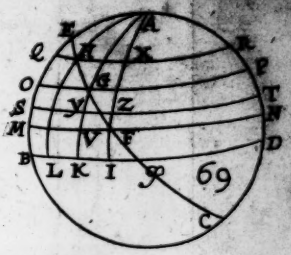

80

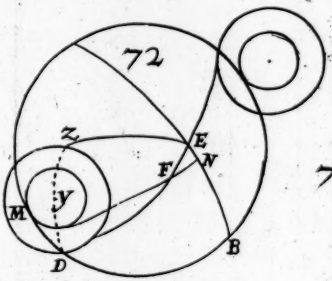

81

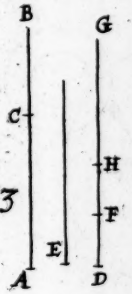
66

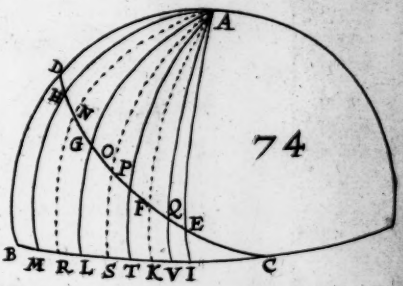

67


68

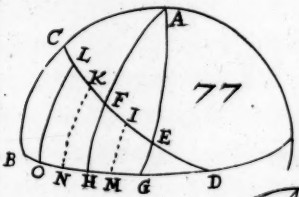

69

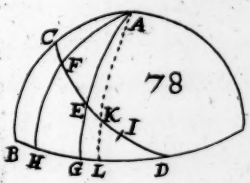

72


73

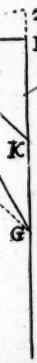

74

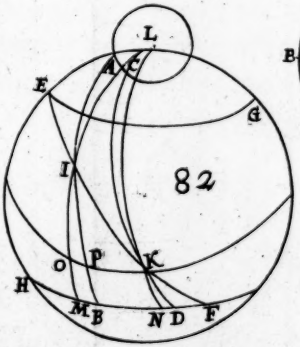
76

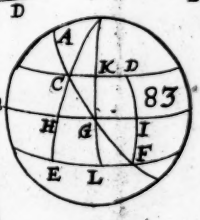

77

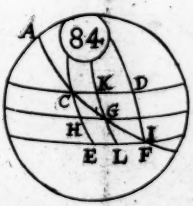

78

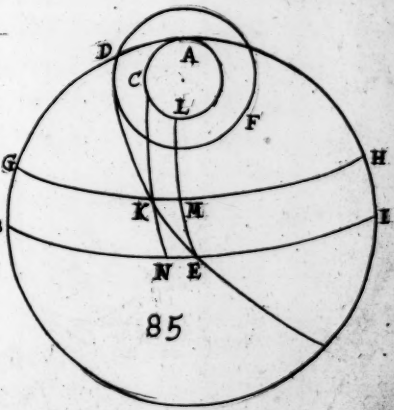
79




82


83


84


85